DOCUMENT RESUME

ED 037 553 VT 010 285

TITLE Demographic Techniques for Manpower Planning in Developing Countries.
INSTITUTION Bureau of Employment Security (Dept. of Labor), Washington, D.C.
SPONS AGENCY Agency for International Development, Washington, D.C.
PUB DATE Apr 66
NOTE 261p.

EDRS PRICE EDRS Price MF-$1.00 HC-$13.15
DESCRIPTORS Bibliographies, Case Records, *Demography, *Developing Nations, Labor Force, *Manpower Needs, *Population Trends, Statistical Data, Tables (Data), *Techniques

ABSTRACT

This handbook explains major demographic techniques which can be used by developing countries to estimate total population and to make projections of population changes. The data were devised for field analysts who have limited mathematical and statistical backgrounds. Anyone with a knowledge of addition, subtraction, multiplication, division, and some algebra will be able to use the methods in the handbook. Several methods for estimating demographic techniques are presented. The choice of the particular method depends upon the availability and reliability of basic data, time involved in undertaking different computations, mathematical and statistical skill of the investigator, and the level of reliability prescribed for a particular estimate. A case study on the Fiji Islands and a selected bibliography are included in the report. (BC)

GM-8 C1

ED037553

DEMOGRAPHIC TECHNIQUES FOR MANPOWER PLANNING IN DEVELOPING COUNTRIES

MAY 1963
Revised APRIL 1966
Reprinted 1968

DEPARTMENT OF STATE
AGENCY FOR INTERNATIONAL DEVELOPMENT
OFFICE OF LABOR AFFAIRS
WASHINGTON, D.C. 20523

VT010285

ED037553

DEMOGRAPHIC TECHNIQUES FOR MANPOWER PLANNING IN DEVELOPING COUNTRIES

Prepared by

The United States Employment Service

Bureau of Employment Security

U. S. Department of Labor

for

THE AGENCY FOR INTERNATIONAL DEVELOPMENT

First Printing May, 1963
Reprinted October, 1963
Revised April, 1966
Reprinted 1968

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

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Demographic Techniques For Manpower Planning In Developing Countries

FOREWORD

Important for the economic development process of any nation is an accurate assessment of its manpower. Such an assessment will set both the dimensions and goals for realistic economic planning recognizing the limitations imposed by the makeup of the labor force immediately available and that expected to exist as the economy develops. Consideration of the future manpower situation involves an understanding of those changes which can be anticipated in the actual labor force and in those portions of the population closely related to the labor force which hold the key to its future expansion.

In developing a series of handbooks for Manpower technicians, the Department of Labor recognized the need for simplified techniques for estimating the labor force and that part of the population which might be affected by manpower development programs. This handbook, Demographic Techniques For Manpower Planning In Developing Countries, was prepared to meet this need. It was written by Riad Tabbarah, then with the Tennessee Department of Employment Security, and now Social Affairs Officer, Department of Economic and Social Affairs, United Nations. It was prepared in the U. S. Employment Service, Louis Levine, Director, under the immediate supervision of V. D. Chavrid. The author acknowledges with appreciation the assistance given in the initial preparation by Dr. E. J. Eberling, Chief of Research, and Mr. John Smith of the Tennessee Department of Employment Security and by Mr. Denis F. Johnston, Bureau of Labor Statistics, U. S. Department of Labor for his review of the original draft. Final review and preparation of the finished handbook for printing was by John T. Murray, Allan E. Broehl and Linda Wright of the Division of International Manpower Assistance, United States Employment Service, Bureau of Employment Security. The American Journal of Hygiene kindly permitted the reproduction of tables developed by Lowell J. Reed and Margaret Merrell for their article, "A Short Method for Constructing an Abridged Life Table", American Journal of Hygiene, Volume 30, 1939.

The aim of this handbook is to explain some of the major demographic techniques with special attention to their application to the developing countries. It is devised mainly for field analysts with limited mathematical and statistical background. To the mathematically minded demographer this handbook may be entitled "demography made simple"--at times, perhaps, a little too simple. Mathematical and statistical jargon has in some instances been sacrificed in favor of more literary exposition and language. Also for the sake of simplicity some demographic concepts have been somewhat oversimplified.

Yet, in order to understand the techniques expounded, the uninitiated needs to work out the examples and illustrations very carefully. He may, moreover, find it necessary to read this book in the presented sequence of topics and not move to the next topic before the one at hand is clearly understood. We would nevertheless like to believe that in order to understand the techniques expounded in this handbook, even the uninitiated analyst will need to be equipped with only a few basic requirements -- an adequate knowledge of addition, subtraction, multiplication, and division; preferably also a knowledge of the most fundamental principles of algebra; and, finally, a good deal of perseverance.

This last requirement is, perhaps, the most crucial. Many of the less mathematically inclined analysts may throw away a handbook at the first sight of a mathematical formula. To them the whole field of mathematics is a mysterious, frightening animal. But with a little perseverance, even these analysts will find that the mathematics used in this handbook is, after consideration, extremely simple if not slightly interesting. In order to reach this stage, one will have to work through the different computations very carefully.

In the following pages the reader will find alternative methods of estimating and projecting population and the labor force in totals and by major demographic characteristics. It is difficult, if not impossible, to determine *a priori* what method an investigator should use in a particular situation and it is this reason in particular that prompted the exposition of different techniques. Let us hasten to add that the techniques expounded are by no means exhaustive. The established techniques that are not discussed by far exceed those that are. We have selected, combined, and in some instances devised, methods that require basic data generally available in the developing nations.

Generally, the choice of method among the alternatives offered in this handbook should be determined after due consideration is given to the following:

First, the availability and reliability of the basic data used in the different methods is of primary importance. Obviously, a method requiring the use of data that are completely unreliable or non-existent is perfectly useless irrespective of its inherent consistency. In such instances, a less accurate method may prove to be of greater practical value.

Second, the time involved in undertaking the different computations may, in some cases, be a major factor in the investigator's choice of method. An estimator may thus be willing to sacrifice some accuracy and reliability in the final estimate for the sake of saving time in obtaining the desired estimate. A less laborious and reliable method may then be preferred to a more involved and sophisticated one.

Third, the mathematical and statistical skill of the investigator may, in some cases, prove to be of major significance in his choice of method. The field of demography is heavily spotted with mathematics and statistics and, up to a certain point, it seems generally true that the more reliable the method, the more highly technical the computations (though not necessarily vice-versa). Major and minor adjustments of basic data do not usually figure in the rougher methods while they may very well constitute a major part of the more reliable ones.

Finally, a major factor which should be considered before a choice of method is made is the level of reliability prescribed by the purpose of the estimate. For some purposes, only the crudest estimates may be required while for other purposes only the most refined may be deemed acceptable.

Before embarking on the careful reading of this handbook, if this is at all the intention of the reader, may we advise the careful reading of the Table of Contents. The reader, we feel sure, will find less difficulty in following the sequence of the argument if he refers to this table after every chapter.

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Demographic Techniques For Manpower Planning In Developing Countries

PART I

ESTIMATING AND PROJECTING TOTAL POPULATION

Censuses, Counts, And Estimates

Until recently, the main function of a census was to determine the total number of people in a community or country. The difference between a population "count" and a population "census" was, therefore, of minor significance. Only since the turn of the century have censuses been designed to furnish information about the social and economic characteristics of people as well as the total number. Presently, information about the size of a population (population count) is only a minor aspect of most censuses.

Except by accident, the exact number of people living in a certain area or country at a given date can only be secured through a complete count of these people at that date. However, even in the most economically advanced countries, periodic population counts (as part of a census) contain inaccuracies. Some people are counted twice (double counting) and usually more are missed completely (undercounting).

A census count of population is, therefore, never completely accurate. It is only an approximation of total population. However, a well planned and executed census remains the most reliable method for approximating the real size of a population.

There are several reasons why estimates of total population are, nevertheless, often made and used instead of direct counting. In the first instance, a carefully planned and executed count of a population is relatively costly to undertake. While the information concerning the total number of people might be deemed useful on an annual basis, the cost involved in undertaking a full count makes such a yearly undertaking somewhat prohibitive. This is especially true when one considers the relatively low cost of making the corresponding estimates. While the results of estimates are necessarily less reliable than those of a well devised and executed total count, they are usually accurate enough for most purposes.

Let us hasten to add that sufficiently reliable estimates have ultimately to be based on census counts. Usually, the longer the period over which estimates have to be made, the less reliable these estimates become. Moreover, census counts not only improve the quality of the estimates, but also present a check on them and a framework within which they can be made. The elapsed period between total counts should, therefore, be determined by an appropriate balance between the cost of the count on the one hand, and the cumulating loss of accuracy of estimates over time on the other. In most developed countries, this period, in normal times, has been set at between five and ten years.

Another reason why estimates of total population are sometimes used instead of counts is the often pressing need for information about the number of people at a future date. Planners in all fields of human activities often demand information about the size of the population at a certain date years in advance. Obviously, a population count cannot, in itself, indicate the future size of the population. Only estimates based on census counts can fulfill this task adequately.

Finally, some investigators are interested not only in the actual size of a population but also in its probable size if certain conditions had prevailed. For example, it might be useful to know what the present size of the population of a certain country that experienced a recent epidemic would have been in the absence of this pestilence in order to determine the impact such conditions had on population growth. Again, a population count in itself is not sufficient and an estimate using the appropriate assumptions will have to be made.

In brief, population counts and, more generally, censuses have become an indispensible element in modern societies. They are, however, costly. If undertaken periodically, they may be supplemented with estimates and other devices, such as sample surveys, which are relatively inexpensive. Insofar as population characteristics are concerned, a sample survey taken by fewer but better trained enumerators than those used for a census may provide more reliable data than the census.

Estimates further supplement censuses when additional information is needed. Their flexibility, inherent in their alternative assumptions, make estimates an important source of information on which to base the manpower plans of developing countries.

PART I

CHAPTER I

ESTIMATING TOTAL POPULATION BY EMPIRICAL METHODS

As mentioned above, the most reliable estimates of total population ultimately are based on one or more census counts. It is possible, however, to make "rough" estimates of total population even in the absence of population counts. Estimates based on data other than total population counts may be called "empirical."

Examples of empirical estimates can easily be found in the population literature. 1/ In their simplest form, they involve two separate elements: "basic datum" and a "multiplier." The basic datum pertains to a variable which is assumed to be in a given relation to total population while the multiplier is an estimate of this relation. By multiplying the basic datum by the multiplier, an estimate of total population is secured. If we designate total population by P, the basic datum by D, and the multiplier by M; the following relationship may describe the empirical approach: $P = D \times M$.

For example, the datum may be the area of inhabited land in a given country (say, 10,000 square miles) and the multiplier the estimated density based, perhaps, on a knowledge of the density of a small "typical" portion of the total area (say, 600 per square mile). A rough estimate

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1/ See, United Nations Methods of Estimating Total Population for Current Dates, ST/SOA/Series A, Population Studies, No. 23., pp. 10-24.

of total population could be secured by application of the above formula. Total population (P) is equal to the area of inhabited land (D) multiplied by the estimated density (M); hence: $P = D \times M$ or,

$$P = 10,000 \times 600 = 6,000,000$$

Empirical estimates may involve more than one multiplication process. For example, the Southern Nigeria administration applied the following method: 1/

> In estimating the population of towns, an Officer should estimate the average number of persons in a house and the average number of houses in a compound; the the number of compounds is easily ascertainable and consequently an approximate estimate can be made of the population. When the population of one or more towns has thus been ascertained, it is easy for an Officer to make a "preliminary estimate" of the population in a place through which he travels for the first time.

While the results in this specific instance were branded "a failure", it was admitted that this was due mainly "to uncritical compilation of . . . reports that were available." It was added that "a critical examination of available local reports might have yielded better estimates for those areas concerning which local reports were clearly faulty or deficient, and an improved estimate might have been made for the country as a whole." 2/

Errors in empirical estimates may arise from both the datum and the multiplier. However, "the greatest difficulty usually attaches to the selection of a plausible multiplier, and this is the main reason why (empirical) estimates are usually of a low order of reliability." 3/

1/ R. Kuczynski, Demographic Survey of the British Colonial Empire, Vol. I, pp. 582-586, quoted in Ibid. p. 12.

2/ Ibid.

3/ United Nations, Methods of Estimating Total Population For Current Dates, op. cit., p. 10

Generally speaking, therefore, empirical methods should be used only when the available data are not sufficient for the application of any of the alternative methods given in the following sections.

It is impossible in the limited space available in this publication to state and analyze all, or even most, empirical methods used in the different developing countries. However, the reader might find the following general classification and explanation of selected types of empirical methods useful in most instances. This classification is presented in terms of the types of basic data used in making empirical estimates.

Vital Statistics: Births

Basic Datum	Multiplier
Number of births within estimate year	The inverse of the estimated birth rate

If birth records are kept for the estimate area or country, they would probably contain a certain degree of under-registration. An estimate of completeness of registration should be made. 1/ Total births could then be estimated by multiplying the number of registered births by the inverse of the estimated completeness of registration. For example, if the number of registered births is set at 17,000 with a completeness of registration estimated at 0.80 (or 80 percent), then total births (B) may be estimated thus:

$$B = 17,000 \times \frac{1}{0.80} = 17,000 \times 1.25 = 21,250$$

1/ See Chapter E of Part VI

Once recorded births are adjusted to under-registration, one has to secure an estimate of the birth rate. If no recent past estimates are available for the area or country in question, it is still possible to make such estimates by selecting an area or country where conditions affecting birth rates can be assumed to be similar to those existing in the estimating area. 1/

If in our example the birth rate is estimated at 0.026 (or 26.0 per thousand), the total population (P) can be estimated thusly:

$$P = 21,250 \times \frac{1}{0.026} \quad \text{or,} \quad P = 21,250 \times 38.46 \quad \text{hence:} \quad P = 817,275$$

The exact figure of 817,275 would not warrant publication, however, because of the inexactness of the approximation process. A more appropriate estimate would be "about 800,000."

Vital Statistics: Deaths

Basic Datum	Multiplier
Number of deaths within estimate year	The inverse of the estimated death rate

The procedure here is similar to that demonstrated for birth data. Methods of estimating completeness of registration of deaths are generally the same as those of births. 2/

1/ For estimates of birth rates for countries around the world between 1949 and 1958, see United Nations Demographic Yearbook, 1959, Statistical Office of the United Nations, New York, 1959. For earlier years, consult Ibid., Demographic Yearbook, 1954.

2/ Statistics and estimates of deaths and death rates for different countries around the world may be secured from United Nations, Demographic Yearbook, 1951, and Demographic Yearbook, 1957, Statistical Office of the United Nations, New York; 1951 and 1957, respectively.

Tax Payers and Benefit Recipients: Tax Payers

Basic Datum	Multiplier
Number of Persons paying a given tax in a given fiscal year	One plus the assumed or estimated number of non-paying persons to each tax payer

To illustrate this method, an actual application of it by the German administration in Togoland in 1912-13 is here described:

The number of persons paying taxes on huts in Togoland was known to the authorities to be approximately 25, 260 (basic datum). On the other hand, "vaccination of the population in the entire District, offered an opportunity for counting the huts and the natives in a large part of the District. This showed that on an average, 4.4 inhabitants have to be reckoned for one person subject to taxation." 1/ Hence, the multiplier was arrived at by adding one to 4.4 thus getting a multiplier of 5.4.

By application of the formula: $P = D \times M$, the total population of Togoland was estimated at:

$P = 25,260 \times 5.4 = 136,400$ inhabitants.

Tax Payers and Benefit Recipients: Benefit Recipients

Basic Datum	Multiplier
Number of persons receiving a given benefit	One plus the assumed number of non-receiving persons to each benefit recipient

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1/ R. Kuczynski, The Cameroon and Togoland, Oxford University Press, London, New York, Toronto, quoted in United Nations Handbook, Vol. 1, p. 22.

The procedure here is generally similar to that using taxpayers as basic datum. Recipients of benefits are persons receiving free government aid such as rations and pensions. Their number may constitute the basic datum needed. By questioning these participants at the time they appear for benefits or by other means (i. e., a random sample of recipients at their homes), the number of non-recipients that have to be reckoned for each participant may be determined. The multiplier can thus be estimated by adding one to ratio. The total population can then be estimated by direct application of the formula for empirical estimates, namely, P = D x M.

Partial Counts and Estimates

Basic Datum	Multiplier
Count or estimate of a given age, sex, ethnic, or social group	One plus the estimated number of persons outside the counted group to each member in the group

The basic datum may be the number of persons of some given characteristic, say age, sex, color, occupation, etc. --that is secured through direct count or a reliable estimate. The Multiplier is secured by adding one to an estimate of the number of persons to be reckoned for each counted individual in the manner illustrated above when only one other characteristic is given.

Where more than one characteristic is given, the following method used by the German administration in a particular District in Cameroon should suffice in explaining the procedure. It was reported as follows:

"The number of men has been ascertained through direct enumeration [Basic datum] . . . . . The number of women and children has been calculated by means of a table which, to judge from sample tests, may be considered as more or less accurate. [Multiplier]

"This table provided that the number of women and the number of children were to be calculated by multiplying the number of men for four tribes by 1.25 and by 1.5, respectively, and for the other three tribes by 1.5 and 2, respectively." [1] Thus, in the formula $P = D \times M$, M would equal $1 + 1.25$ (proportion of women) + 1.5 (proportion of children) = 3.75.

In such instances, an estimate of the multiplier requires information concerning the age and/or sex distribution of the population. In case no reliable estimates are available, a country which is believed to have similar birth and migration rates may form the basis for estimating the age and sex proportions in the area or country of interest." [2]

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[1] Ibid., quoted in United Nations Methods of Estimating Total Population for Current Dates, op. cit., p. 21.

[2] For age and sex distribution in different countries around the world, see United Nations Demographic Yearbook, for the years 1955, 1959, and 1960.

PART I

CHAPTER II

ESTIMATING TOTAL POPULATION BETWEEN TWO GIVEN DATES BY INTERPOLATION

In this chapter, the concern will be with estimating total population at one or more years, situated between two points in time, at which reliable counts or estimates of total population are available. The basic technique employed for this purpose will be that of interpolation. Generally speaking, "interpolation is the process of estimating values of a variable quantity between given values, or of locating a point on a curve between given points." 1/ As applied here, interpolation is the process used to estimate total population at any or all years between two given counts or estimates.

It should be obvious that a variable may assume a wide variety of values between two given points. In other words, the trend a variable may follow between two points could, in general, be characterized by wide fluctuations. Fortunately, however, this is not generally the case with population growth. In most instances, the size of a population changes only gradually.

For a population to follow a widely fluctuating rate of growth between two points in time, sharp changes in factors affecting this growth have to take place during the period. War, pestilence, famine, and other such disasters constitute the most powerful of all factors. Because of the unpredictability of such events, they are usually assumed away in

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1/ F. C. Mills, Statistical Methods, Revised Edition, Henry Holt and Co., New York, 1939, p. 70 n.

most projections of population into the future. However, when the estimator is faced with the problem of tracing the path of population growth between two points in time already realized--and this is the concern of this chapter--disasters of the sort just mentioned have to be reckoned if any took place during the period in question. While population may be assumed to have otherwise followed a somewhat stable path between the two given points, allowances have to be made for any turbulent years within the period. For example, if one is faced with tracing the trend of population growth in Germany between 1940 and 1950, one may not assume a steady rate of growth between these two points. Instead, allowance for population losses due to the war has to be made for the first five years of the decade.

Another set of factors affecting the rate of population growth may be classified as economic. A relatively sharp rise or decline in wage levels or the sudden realization of an unusually high or low rate of expansion in economic activities and income in a community or country are cases in point. Such economic factors may affect population growth through two different channels--by influencing the rate of natural increase (births less deaths) and by influencing the direction and extent of migration. Let us note that, when dealing with large populations over short periods of five or even ten years, the effects of changes in the economic determinants on population growth are relatively minor. They present, however, a serious challenge to the interpolator faced with tracing the path of population growth in a small area open freely to in and out migration. In such instances it is highly recommended that allowances be made for such changes based on a careful study of the changes in the economic determinants and their effects on the rate of population growth. In most cases, the estimator is required to use a great deal of value judgment.

The extent and direction of the allowances made for violent shifts in the economic and non-economic determinants of population growth are impossible to determine *a priori*. They depend on the nature and magnitude of the shifts. With the usual lack of adequate statistics on such events, their estimated effect on the rate of population growth is, to a great extent, left to the estimator. It has been found advisable, in many instances, to make full counts during or upon termination of a disaster or an abnormal economic condition. 1/ In the same manner as countries, communities and even firms take account of their natural and capital resources after a major disturbance, so may they take due account of their manpower resources.

In view of the fact that population growth usually follows a somewhat steady pattern over short periods of time (say, five or ten years), allowances for major disturbances from such a pattern cannot be made *a priori*. For the sake of simplicity, the two interpolation techniques discussed in this chapter are both of the "straight line" variety. Arithmetic interpolation, the simpler of the two, assumes a constant yearly *increment* of change between the two given values. Starting with the first given population size, the size of the population at each of the succeeding years is arrived at by adding (or subtracting) the same number of persons to (or from) the previous year's population. If the resulting values are plotted on plain graph paper, where population size is represented on the vertical axis and the respective years on the horizontal axis, the line connecting the different points will be a straight line.

Simple geometric interpolation assumes not that the yearly *increment* to population is the same but that the yearly *rate* of increase is constant between the two given values of total population. By adding *constant* yearly *increments* to a *growing* population, the yearly *rate* of population growth will obviously be falling. On the other hand, a constant positive yearly rate of growth implies larger and larger yearly increments. If the annual population estimates, assuming a constant positive rate of growth between the two given points, are plotted on plain graph paper, they can be joined only by a curve convex from below. However, if such a series is plotted on semilogarithmic paper, the line joining the different points will be a straight line.

1/ A number of states in the U.S. conducted censuses during and after the second World War.

In brief, arithmetic interpolation between two points leads to a straight line trend on plain graph paper while simple geometric interpolation prescribes a straight line on semilogarithmic paper. They are, therefore, both misleadingly called "straight line" interpolations.

Arithmetic Interpolation

Arithmetic interpolation involves two steps:

1. Determination of the yearly increment and
2. Specific application of the yearly increment to the original population to obtain an estimate of total population at an intermediary date.

The population count in Basutoland (a British territory in Africa) on July 5, 1946 showed a total population of 561,000 people. On April 4, 1956 (9 3/4 years later), a similar count revealed a total population of 634,000. Through arithmetic interpolation one can make estimates of total population of Basutoland for any date falling between the two given counts.

Designate the size of the population as reported in the first count by P_o and in the second count by P_n; and designate the period, in years, that elapsed between the two counts by t (in this case 9 3/4 years); then the yearly increment Y can be secured by direct application of the following equation:

$$Y = \frac{P_n - P_o}{t}$$

In our example:

$$Y = \frac{634,000-561,000}{9.75} = \frac{73,000}{9.75} = 7,487$$

The estimated population at any given date (P_x) can be calculated by applying the following equation:

$$P_x = P_o + x(Y)$$

Where x is the number of years between the date of the count and the estimate date; and Y the yearly increment.

For instance, in making an estimate of total population in Basutoland at exactly three years after the first count, one can proceed as follows:

$$P_3 = 561,000 + 3(7,487) \text{ or } P_3 = 561,000 + 22,461 = 583,500$$

Therefore: The estimated population of Basutoland on July 5, 1949 (rounded to the nearest 100) is, by arithmetic interpolation, 583,500 people.

The estimated increment can be checked by multiplying the annual increment by 9 3/4 years, adding the result to the count of July 5, 1946 and the product should be equal to the 1956 count of 634,000. Thus:

$$P_{9.75} = 561,000 + 9.75\,(7,487) \text{ or } P_{9.75} = 561,000 + 72,998$$

Population April 5, 1956 = 634,000. 1/

Simple Geometric Interpolation

Like arithmetic interpolation, geometric interpolation involves two major steps:

1. Determination of the yearly rate of population growth between the two given dates, and
2. Specific application of that rate to the original population to obtain an estimate of total population for an intermediate date.

1/ The exact addition amounts to 633,998. This difference of two persons is due to rounding.

Population change is a continuous phenomenon. Population changes not only every year but also every month, every week, in fact, every instant. On the assumption that it increases between the two given dates at a constant rate, population increases in much the same manner as capital increases when invested at a given rate of interest compounded continuously.

The equation for computing total capital for any given period can be utilized for determining population changes and is as follows:

$$C_n = C_o e^{Rt}$$

Where C_o = the original capital (or starting population count),

R = rate of interest (rate of change),

t = time over which the change took place,

C_n = capital at end of t years (ending population count),

e = conversion factor which relates normal or Naperian logarithms to common logarithms which are generally available. This has a constant value of 2.7813.

Applied to population growth, the equation becomes:

$$P_n = P_o e^{Rt}$$

For computational simplicity, this can be written in logarithmic form:

$$\log P_n = \log P_o + Rt \ (\log e) \quad \text{or} \quad R = \frac{\log \left(\frac{P_n}{P_o}\right)}{t \log e}$$

Note:

If normal or Naperian logarithmic tables are used, the equation would be:

$$R = \frac{\log \left(\frac{P_n}{P_o}\right)}{t}$$

Since log e = 0.4343, the equation can be written:

$$R = \frac{\log \left(\frac{P_n}{P_o}\right)}{t\ (0.4343)}$$

In the example of Basutoland, the equation would read:

$$R = \frac{\log \left(\frac{634,000}{561,000}\right)}{(9.75)\ (0.4343)} = \frac{\log\ (1.130)}{4.2344}$$

From a table of common logarithms, the value of log (1.130) is determined to be: 0.05308

Hence:

R = 0.0125 or 1.25 percent a year.

First arithmetic approximation (r_1) of the geometric rate (R).

Some analysts may find the computation of R either inconvenient or unnecessarily involved. This is justified when either a table of common logarithms is not available, or the available population counts are so inaccurate that accuracy in determining R is not absolutely necessary.

In such a circumstance R can be approximated by following more simple arithmetic methods. The following illustrates three possible methods.

The formula giving the first arithmetic approximation (r_1) of the annual rate of change (R) is the following:

$$r_1 = \frac{\frac{P_n}{P_o} - 1}{t}$$

In the Basutoland example:

$$r_1 = \frac{1.130 - 1}{9.75} = \frac{0.130}{9.75}$$

hence:

$r_1 = 0.0133$ or 1.33 per cent a year.

Second arithmetic approximation (r_2) of the geometric rate (R).

A much better approximation of R can usually be achieved by direct application of the following equation:

$$r_2 = \frac{I_a}{P_a}$$

where:

$$I_a = \frac{P_n - P_o}{t}$$ (The yearly increment in arithmetic interpolation) and

$$P_a = \frac{P_n + P_o}{2}$$

In our example:

$$I_a = \frac{73,000}{9.75} = 7,487$$

and

$$P_a = \frac{1,195,000}{2} = 597,500$$

hence:

$$r_2 = \frac{7,487}{597,500}$$

$$\underline{r_2 = 0.0125 \text{ or } 1.25 \text{ per cent a year.}}$$

A further approximation (r_3) of the geometric rate (R).

Drawing again on formulae used in computing interest on capital, it is found that, in many instances, interest is compounded annually, semi-annually, or a specified number of times in a given year. Also, annual compounding over a short number of years gives a series of yearly capital values that are not too different from those given by continuous compounding. Looking at it in a slightly different way, one may conclude that the rate (r_3) arrived at by the annual compounding technique could be used as an approximation of the geometric rate (R) as computed through the continuous compounding formula. As an illustration, this technique is applied to the example for Basutoland.

The formula used in annual compounding and adapted to population is the following:

$$P_n = P_o (1 + r_3)^t$$

hence:

$$r_3 = \sqrt[t]{\frac{P_n}{P_o}} - 1$$

For computational simplicity use:

$$\log (1 + r_3) = \frac{\log \left(\frac{P_n}{P_o}\right)}{t}$$

Therefore, from the example:

$$\log (1 + r_3) = \frac{\log 1.130}{9.75}$$

or

$$\log (1 + r_3) = \frac{0.05308}{9.75} = 0.00544$$

In a table of common logarithms, it is found:

$0.00518 = \log 1.012$

$0.00561 = \log 1.013$

and by interpolation: $(.001 : x = .00043 : .00017 = .0004)$

antilog of $0.00544 = \log 1.0126$

or: $1 + r_3 = 1.0126$

hence:

$r_3 = 0.0126$ or 1.26 percent a year, a very close approximation of

$R = 1.25$ percent a year.

Application of R

Once R or an approximation of it is found, the second and final step is to estimate the population at a given date between the two counts using this rate.

The formula to apply in this instance is that of continuous compounding, namely,

$$P_x = P_o e^{Rx}$$

Where P_x = population after x years from first count;

P_o = population at first count;

e = a constant - 2.7183;

and R, the rate of population growth.

For computational purposes use:

$$\log \left(\frac{P_x}{P_o}\right) = Rx \log e \qquad \log e = 0.4343$$

Then obtain:

$$P_x = (\text{antilog of } Rx \log e) \; P_o$$

As mentioned earlier, the annual compounding formula, which is somewhat simpler to use, offers an approximation of the continuous compounding formula. R, or an approximation of it, can hence be substituted in either formula to obtain an estimate of total population between the two given dates. The annual compounding formula, it must be remembered, is the following:

$$P_x = P_o (1 + r)^x$$

For computational purposes, it might be more convenient to use the corresponding logarithmic functions:

$$\log \left(\frac{P_x}{P_o}\right) = x \log (1 + r)$$

then:

$$P_x = [\text{antilog of } x \log (1 + r)] \; P_o$$

In the following, R is applied in both compounding formulae to obtain estimates of total population in Basutoland for July 5, 1949, and to obtain, in tabular form, yearly estimates of total population arrived at by arithmetic interpolation and by each of the eight simple geometric interpolation methods stated above.

Continuous Compounding

It is known that:

$$R = 0.0125, \; P_o = 561,000 \text{ and } x = 3$$

Hence, by substituting the values of P_o, R, x, and log e in the right side of the continuous compounding equation (page 16), the following is obtained:

$$\log \left(\frac{P_3}{P_o}\right) = 0.0125 \;\; (3) \;\; (0.4343) = 0.01629$$

From a table of common logarithms of numbers:

antilog of 0.01629 = 1.0382

Hence, by substituting in the succeeding equation

$$P_3 = 561{,}000\ (1.0382), \text{ hence:}$$

$$P_3 = 582{,}400$$

Annual Compounding

Using the same R = 0.0125 in the annual compounding formula (page 19) in its logarithmic equivalents, 1/ it is found that:

$$\log\left(\frac{P_3}{P_o}\right) = 3 \log 1.0125$$

From a table of common logarithms of numbers:

$$\log 1.0125 = 0.00540$$

Therefore:

$$\log\left(\frac{P_3}{P_o}\right) = 0.01620$$

1/ The use of logarithm is less essential in this case. By direct substitution we have:

$$P_3 = 561{,}000\ (1.0125)^3$$

or: $$P_3 = 561{,}000\ (1.0380)$$

hence:

$$P_3 = 582{,}300$$

however:

When x is greater than 5, it becomes quite laborious to compute the value of $(1 + r)^x$.

From a table of common logarithms, it can be determined that:

antilog of 0.01620 = 1.0380

Substitute to obtain:

$$P_3 = 561,000 \times 1.0380$$

Hence:

$\underline{P_3 = 582,300}$; a very close approximation of $P_3 = 582,400$ arrived at by continuous compounding formula.

In the preceding section, it was explained that two steps are involved in making population estimates between two given counts by simple geometric interpolation; namely, finding the annual rate of population growth (R) between the two points or an approximation thereof (r_1, r_2, or r_3) and applying this rate to one of two formulae. Theoretically, therefore, eight different estimates of total population for any one year during the given period can be made by simple geometric interpolation. In practice, however, some approximations of R (probably r_2 and r_3) might very likely give the same figure as R to four decimal places so that, ultimately, only four or even less different estimates for any single year are obtained. Even these should, in most instances, be very close to each other.

The following table (Table I) indicates yearly population estimates for the Basutoland population computed following the different methods described in this chapter. To make estimates of each year's population, a well organized worksheet should prove to be helpful. Two sample worksheets (Worksheets 1 and 2) follow the table; one using the continuous compounding formula and the other the annual compounding formula.

Demographic Techniques For Manpower Planning In Developing Countries

Summary

If no major disturbances in the natural and economic determinants of population growth took place between two given counts or estimates, interpolations of the straight line variety become adequate for estimating total population at given dates within the period. The simplest of all interpolation techniques is the arithmetic which assumes that population increased by constant yearly increments over the period. A slightly more involved technique is that of geometric interpolation which assumes that the size of a population increases at a constant yearly rate between the two dates.

Both interpolation techniques involve two general steps. In arithmetic interpolation, it is first necessary to determine the yearly increment-- and then to apply this increment to a simple formula.

Geometric interpolation involves the determination of the rate of population growth between the two dates and the application of this rate to a formula to obtain an estimate of total population at a given date between the two points.

TABLE I

YEARLY ESTIMATES OF BASUTOLAND POPULATION BETWEEN 1946 AND 1956 FOLLOWING DIFFERENT INTERPOLATION METHODS (IN THOUSANDS)

Date	Arithmetic Interpolation	Geometric Interpolation							
	Increment = 7.487	$R = 0.0125$		$r_3 = 0.0126$		$r_1 = 0.0133$		$r_2 = 0.0125$	
		C. C. *	A. C. **	C. C. *	A. C. **	C. C. *	A. C. **	C. C. *	A. C. **
7/5/46*** (count)	561.0	561.0		561.0		561.0		561.0	
7/5/47	568.5	568.1	568.0	568.1	568.1	568.5	568.5	568.1	568.0
7/5/48	576.0	575.2	575.1	575.3	575.9	576.1	576.0	575.2	575.1
7/5/49	583.5	582.4	582.3	582.6	583.2	583.8	583.7	582.4	582.3
7/5/50	590.9	589.8	589.6	590.0	590.7	591.6	591.5	589.8	589.6
7/5/51	598.4	597.2	597.0	597.5	598.4	599.6	599.3	597.2	597.0
7/5/52	605.9	604.7	604.5	605.0	606.2	607.6	607.3	604.7	604.5
7/5/53	613.4	612.3	612.1	612.7	614.1	615.8	615.4	612.3	612.1
7/5/54	620.9	620.0	619.7	620.5	622.1	624.0	623.6	620.0	619.7
7/5/55	628.4	627.8	627.4	628.4	630.2	632.4	631.9	627.8	627.4
4/4/56	634.0	633.7	633.3	634.4	636.2	638.7	638.2	633.7	633.3
4/4/56 (count)***	634.0	634.0		634.0		634.0		634.0	

* Continuous Compounding

** Annual Compounding

*** Census figures, not estimates

WORKSHEET I

YEARLY ESTIMATES BASUTOLAND POPULATION BY INTERPOLATION USING R=0.0125 AND THE CONTINUOUS COMPOUNDING TECHNIQUE

1946 - 1956

Date	Years (x)	R	Rx	Rx log e	antilog of (Col. 4)	Po (in 000's)	Px (in 000's)
			(1) x (2)	(Col. 3)x(0.4343)			(5) x (6)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
7/5/47	1	0.0125	0.0125	0.00543	1.0126	561	568.1
7/5/48	2	0.0125	0.0250	0.01086	1.0253	561	575.2
7/5/49	3	0.0125	0.0375	0.01629	1.0382	561	582.4
7/5/50	4	0.0125	0.0500	0.02172	1.0513	561	589.8
7/5/51	5	0.0125	0.0625	0.02714	1.0645	561	597.2
7/5/52	6	0.0125	0.0750	0.03257	1.0779	561	604.7
7/5/53	7	0.0125	0.0875	0.03800	1.0915	561	612.3
7/5/54	8	0.0125	0.1000	0.04343	1.1052	561	620.0
7/5/55	9	0.0125	0.1125	0.04886	1.1191	561	627.8
4/4/56	9.75	0.0125	0.1219	0.05294	1.1296	561	633.7

WORKSHEET 2

YEARLY ESTIMATES OF BASUTOLAND POPULATION BY INTERPOLATION USING $r_1 = 0.0133$ AND THE ANNUAL COMPOUNDING TECHNIQUE

1946-1956

Date	Years (x)	log $(1+r_1)$	Col. 2 x Col. 3	antilog of (Col. 4)	Po (in 000's)	Px (in 000's) (Col. 5) x (Col. 6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
7/5/47	1	0.00574	0.00574	1.0133	561	568.5
7/5/48	2	0.00574	0.01148	1.0268	561	576.0
7/5/49	3	0.00574	0.01722	1.0405	561	583.7
7/5/50	4	0.00574	0.02296	1.0543	561	591.5
7/5/51	5	0.00574	0.02870	1.0683	561	599.3
7/5/52	6	0.00574	0.03444	1.0825	561	607.3
7/5/53	7	0.00574	0.04018	1.0969	561	615.4
7/5/54	8	0.00574	0.04592	1.1115	561	623.6
7/5/55	9	0.00574	0.05166	1.1263	561	631.9
4/4/56	9.75	0.00574	0.05597	1.1376	561	638.2 1/

1/ Note that the estimate obtained here is about 4,000 persons more than the count of that date. This is mainly due to the fact that the rate used (r_1) is only the crudest approximation of the actual rate of population growth during the period and, is usually slightly higher than the actual rate.

PART I

CHAPTER III

PROJECTING TOTAL POPULATION TO FUTURE DATES BY EXTRAPOLATION

The aim of this chapter is to explain the use of extrapolation techniques for estimating total population for future dates. Basically, extrapolation is the process of extending a curve at one end or the other. In the case of time series such as yearly population data, extending the curve beyond the last date of the series may be termed extrapolating into the future (or projecting, forecasting, etc. by extrapolation).

Extrapolating into the future presents a marked similarity with interpolating between two given points. A thorough understanding of the techniques introduced in the previous chapter is, therefore, extremely useful for a good understanding of those used in this one.

In the following pages, some of the simpler methods of extrapolation as applied to population data will be explained. The simplest of these is arithmetic extrapolation. Slightly more complicated are simple geometric extrapolation and what can be termed as the polynomial extrapolation.

Arithmetic Extrapolation

Arithmetic interpolation and arithmetic extrapolation are based on the same general assumption; namely, that the size of a given population changes by constant yearly increments. In the case of arithmetic interpolation, the size of the yearly increment is arrived at by dividing the change in total population between the two counts or estimates by the corresponding number of years. This obviously cannot be done in the case of extrapolation into the future because the size of the population at the end of the period is not known. However, an estimate of the yearly increment may be secured from previous periods. The specific period from which such an estimate is secured is called the base period.

The choice of a base period is a crucial step in making reliable population forecasts. Generally, "any forecasting procedure which involves merely the continuation of a curve or the automatic application of a formula, without at the same time requiring a careful consideration of underlying and modifying factors, is hardly to be depened upon." 1/ In the case of population forecasting, the choice of a base period should involve careful consideration of those underlying factors for it is mainly on that choice that the reliability of the final estimate depends.

The principles that should govern the choice of a base period may be summarized in three general propositions:

> First, it should depend on the purpose of the estimate. If the probable future size of the population is to be estimated, then the base period to be chosen must, after careful consideration, be likely to have had similar trends in the socio-economic factors determining population growth as is expected in the projection period. 2/

1/ F. E. Croxton and D. J. Cowden, Applied General Statistics, Second edition, Fifth printing, Prentice Hall, Inc., Englewood Cliffs, N. J., May, 1960, p. 112.

2/ If such a period does not exist, it may be possible to choose the period with the closest resemblance to what is expected in the projection period and the magnitude of population growth over the former arbitrarily adjusted to fit the latter.

Second, periods of unusual booms or unusual disaster such as wars, pestilences and famines should, in most instances, be ruled out as base periods unless the purpose of the estimate is to find, not the most probable population size, but the most likely size under similar conditions. If the estimator is interested in the most probable size of a population at a future date, unusual conditions cannot be assumed because of their unpredictability. In any case, however, it is best to state explicitly the assumption underlying the choice of the base period.

Finally, when using arithmetic extrapolation, the base period should not be too far distant in the past. If population is growing, the yearly increments of far past periods applied to recent population counts may well result in absurdly low rates of population growth, while if population is declining the rates thus obtained may well be absurdly high. The magnitude of the understatement or overstatement of population grows progressively larger as the period of the projection increases. Because of this, the use of arithmetic extrapolation methods is not recommended for forecasting population estimates for periods exceeding five years from the end of the base period.

Once the base period is determined, the basic mechanics of projecting population by arithmetic extrapolation become similar to those of arithmetic interpolation as explained in the previous chapter. They involve two steps: (1) determining the yearly increment during the base period, and (2) applying the result to a forecasting formula. The basic formulae used in both steps are the same as those used in the corresponding steps of arithmetic interpolation.

To compute the yearly increment (Y) use the equation:

$$Y = \frac{Pn - Po}{t}$$

Where P_n and P_o = the terminal and initial years' population totals found in the base period and t = the number of years covered in the base period.

For the projected total population at a given year use:

$$P_x = P_1 + xY$$

where P_1 = the population at the last count,

x = the number of years between the last count and the year of estimate, and

Y = the yearly increment.

To illustrate the computations in Swaziland, population counts of 1936, 1946 and 1956, were 157,000, 185,000 and 237,000 respectively. An estimate is needed of the total population of Swaziland in 1960 through arithmetic extrapolation.

As explained earlier, after choosing a base period the procedure involves two steps; namely, (1) estimate the yearly increment and (2) apply the result to the forecast formula.

The available data indicate two possible base periods; 1936-1946 and 1946-1956. The first period may be eliminated on at least two counts: First, it contained a war; and second, early counts in Swaziland are most probably less accurate than the more recent ones because, in most instances, great improvements in the quality of counts are made at the early stages of census taking. The period 1946-1956 seems to be more appropriate for a base. More careful consideration of the base period, however, may point out the necessity of making some adjustments in the yearly increment before it is applied to the projection period, 1956-1960. These adjustment should be made on the basis of the analysts best judgment of the changing circumstances.

Again, the formula for the yearly increment is the following:

$$Y = \frac{P_n - P_o}{t}$$

in our example:

$$Y = \frac{P_{56} - P_{46}}{10}$$

or,

$$Y = \frac{237,000-185,000}{10}$$

hence:

$$Y = 5,200$$

Again, from arithmetic interpolation, the formula to be applied is:

$$P_{60} = P_{56} + 4Y$$

or

$$P_{60} = 237,000 + 20,800 = 258,000 \text{ (rounded)}$$

Therefore, the estimated population of Swaziland in 1960, by arithmetic extrapolation, and with 1946-1956 as base, is 258,000.

Simple Geometric Extrapolation

Little needs to be said in way of introducing this section on simple geometric extrapolation because the reader should by now be familiar with its principles. These, again, involve two steps following the selection of an appropriate base period.

1. Finding the yearly rate of population growth (R) or an approximation thereof (r_1, r_2, or r_3), and

2. Applying this rate to either the continuous compounding, or the annual compounding formula.

As in the previous section, estimating the 1960 population of Swaziland is used to illustrate the methodology. Again, the 1936, 1946, and 1956 counts were 157,000, 185,000 and 237,000 respectively.

Because of the reasons stated in the previous section, the base period chosen here is 1946-1956.

The formulae used in computing the yearly rate of population growth (R) and its different approximations, (r_1), (r_2), and (r_3) are the same as in the previous chapter. We shall, therefore, restrict the narrative to a minimum.

R:

$$R = \frac{\log \frac{P_{56}}{P_{46}}}{10 \log e} = \frac{\log \left(\frac{237,000}{185,000}\right)}{10\ (0.4343)}$$

or:

$$R = \frac{\log 1.281}{4.343}$$

hence:

$$R = \frac{0.10755}{4.343}$$

and:

$$\underline{R = 0.0247} \quad \text{or} \quad \underline{2.47 \text{ percent}}$$

First Arithmetic Approximation (r_1) of the Geometric Rate (R)

$$r_1 = \frac{\frac{P_{56}}{P_{46}} - 1}{10} = \frac{1.281-1}{10}$$

and:

$$\underline{r_1 = 0.028} \quad \text{or} \quad \underline{2.8 \text{ percent}}$$

Second Arithmetic Approximation (r_2) of the Geometric Rate (R)

$$r_2 = \frac{I_a}{P_a}$$

Where:

$$I_a = \frac{P_{56} - P_{46}}{10} \quad \text{and} \quad P_a = \frac{P_{56} + P_{46}}{2}$$

hence:

$$I_a = 5,200 \text{ and } P_a = 211,000$$

and:

$$\underline{r_2 = 0.0246} \quad \text{or} \quad \underline{2.46 \text{ percent}}$$

A Further Approximation (r_3) of the Geometric Rate (R)

$$r_3 = \sqrt[10]{\frac{P_{56}}{P_{46}}} - 1$$

by rearranging in log form:

$$\log (r_3 + 1) = \frac{\log 1.281}{10}$$

hence:

$$\log (r_3 + 1) = 0.01076$$

therefore:

$$r_3 + 1 = 1.0250$$

and:

$$\underline{r_3 = 0.0250} \qquad \text{or} \qquad \underline{2.50 \text{ percent}}$$

Continuous Compounding: R = 0.0247

$$P_{60} = P_{56}\, e^{4R}$$

by rearranging in log form:

$$\log \left(\frac{P_{60}}{P_{56}}\right) = 4R \log e$$

or:

$$\log\left(\frac{P_{60}}{P_{56}}\right) = (4)\ (0.0247)\ (0.4343)$$

hence:

$$\log\left(\frac{P_{60}}{P_{56}}\right) = 0.04291$$

so that:

$$\frac{P_{60}}{P_{56}} = \text{antilog of } 0.04291 = 1.1039$$

therefore:

$$P_{60} = 1.1039 \times P_{56}$$

hence:

$$P_{60} = 1.1039 \times 237{,}000$$

and:

$$P_{60} = 262{,}000 \quad \text{(rounded)}$$

Annual Compounding: $R = 0.0247$

$$P_{60} = P_{56}\ (1 + r)^4$$

by rearranging in logarithmic form:

$$\log \left(\frac{P_{60}}{P_{56}}\right) = 4 \log \ (1 + r)$$

hence:

$$\log \left(\frac{P_{60}}{P_{56}}\right) = 4 \log 1.0247$$

therefore:

$$\log \left(\frac{P_{60}}{P_{56}}\right) = 4 \ (0.01031)$$

so that:

$$\log \left(\frac{P_{60}}{P_{56}}\right) = 0.04124$$

hence:

$$\frac{P_{60}}{P_{56}} = \text{antilog of } 0.04124 = 1.0996$$

thus:

$$P_{60} = 1.0996 \times P_{56}$$

or:

$$P_{60} = 1.0996 \times 237,000$$

and:

$$P_{60} = 261,000 \quad \text{(rounded)}$$

As it was said earlier, simple geometric extrapolation, as defined and explained in this handbook, results in eight separate but not necessarily different estimates of total population. This is obvious when one considers the fact that four estimates of the yearly rate of population growth have been derived and that each may be applied to two compounding formulae. In our example of Swaziland, the estimates resulting from these eight different combinations are the following:

a/ Continuous Compounding

for $R = 0.0247$ $P_{60} \cong 262,000$ (computed above)

for $r_1 = 0.0281$ $P_{60} \cong 265,000$

for $r_2 = 0.0246$ $P_{60} \cong 262,000$

for $r_3 = 0.0250$ $P_{60} \cong 262,000$

b/ Annual Compounding

for $R = 0.0247$ $P_{60} \cong 261,000$ (computed above)

for $r_1 = 0.0281$ $P_{60} \cong 265,000$

for $r_2 = 0.0246$ $P_{60} \cong 261,000$

for $r_3 = 0.0250$ $P_{60} \cong 262,000$

Note that the only estimate out of line is that using the roughest approximation of (R) namely, (r_1).

POLYNOMIAL EXTRAPOLATION

A polynomial extrapolation is a projection made with the help of a polynomial function. A polynomial function can be represented by a first degree equation of the form

$$Y = a + bX$$

a second degree equation of the form

$$Y = a + bX + cX^2$$

or any degree thereafter.

In the following, only these two types of equations will be used since higher degree polynomial functions are seldom necessary in short-term population projections.

The First Degree Polynomial

This is considered the simplest of all mathematical functions. The first degree polynomial is descriptive of an arithmetic progression, i.e., a constant absolute increment or decrement per unit of time. It represents, therefore, essentially the same technique used in arithmetic interpolation and extrapolation and the result of its application to population data should form a straight line when plotted on plain graph paper. In fact, its use here only formalizes the principles of arithmetic extrapolation. Some of its major applications to population data are expounded here not only because of this formalization but also as an introduction to the similar but more complicated technique of parabolic extrapolation which uses second degree polynomials.

The reader must have already noted that all mathematical projections require the choice of an adequate base period. The criteria to be considered in choosing the base period are principally the same, irrespective of the type of mathematical projection used. To eliminate repetition, therefore, the reader is asked to consult previous sections for these criteria.

If the values of the two counts are plotted on plain graph paper where time is represented on the x axis (the abcissa) and the size of population on the y axis (the ordinate), a straight line joining the two points may easily be drawn. The values represented along this line, if read properly, should correspond exactly to the values of total population obtained by arithmetic interpolation. Similarly, an extension of the line should indicate values of total population arrived at by arithmetic extrapolation.

While careful free hand drawing of such a straight line, passing through the two given values of total population, is perfectly adequate and may replace the calculations involved in arithmetic interpolation and extrapolation, it is useful, for reasons which will become clear shortly, to introduce at this point a third method for making this type of straight line interpolation and extrapolation. This method employs as its major tool the polynomial of the first degree, namely:

$$Y = a + bX$$

In order to illustrate the mechanics of extrapolating with a first degree polynomial, we shall use the same example used previously in this chapter; that of estimating the 1960 population of Swaziland given the counts of 1946 (185,000) and 1956 (237,000).

The specific first degree polynomial that applies to our example may be obtained as follows:

The number of years which elapsed after the first year of the base period (in this instance, 1946) is given by the value of X, while the corresponding population size is indicated by the value of Y. Hence, by substitution, the equation may be written:

$$185,000 = a + b\ (0)$$

or:

$$237,000 = a + b\ (10)$$

The values of a and b can be obtained by solving these two equations:

$$a = 185,000$$

by substituting

$$237,000 = 185,000 + 10b$$

hence:

$$b = 5,200$$ (= the yearly increment obtained by arithmetic extrapolation above)

Therefore, the equation describing the straight line passing by the two given population values to an estimate (Y) for a future time period X is:

$$Y = 185,000 + 5,200X$$

To estimate the 1960 population in the equation we substitute 14, the value of X, because 1960 falls 14 years after the first year of the base period, thus:

$$Y_{60} = 185,000 + 5,200\ (14)$$

hence:

$$Y_{60} = 258,000$$

It should be noted that this value of 1960 population of Swaziland is exactly the same as that arrived at by arithmetic extrapolation and, furthermore, that it could have been obtained graphically by extending the line joining the two given population values plotted on plain graph paper to correspond to the 1960 point on the x axis and then reading the corresponding value on the y axis.

In case the base period deemed adequate contains more than two counts or estimates, it may be advisable to make straight line projections taking all or few of the given population values into consideration. But in such instances the plotted population values may not, in all likelihood, fall in a straight line path so that it becomes impossible to connect them with a straight line. However, the estimator may draw a line that passes "in the middle" of all the given values for use in projecting total population. More formally, the straight line that gives the best fit is that which is laid out in such a way that the sum of the deviations of the given values from it is equal to zero. Such a line is usually referred to as the line of least squares.

This line may be roughly drawn by free hand but may be more accurately obtained by using a similar method as the one just described.

To illustrate, one can use the example of Puerto Rico where the last four population counts are given in the following table:

TABLE II
POPULATION OF PUERTO RICO
1930, 1940, 1950, and 1960

Year	Population (in thousands)
1930	1, 544
1940	1, 869
1950	2, 211
1960	2, 350

The problem is to find the 1965 population estimated by straight line extrapolation using these four counts.

Therefore:

For 1930 $X = -20$ and $Y = 1{,}544$
For 1940 $X = -10$ and $Y = 1{,}869$
For 1950 $X = 0$ and $Y = 2{,}211$
For 1960 $X = 10$ and $Y = 2{,}350$
For 1965 $X = 15$ and Y_{65} is to be estimated

Hence, by substituting in:

$$Y = a + bX$$

The four following equations are obtained:

$$\begin{aligned} 1{,}544 &= a - 20b \\ 1{,}869 &= a - 10b \\ 2{,}211 &= a \\ 2{,}350 &= a + 10b \end{aligned}$$

In order to fit three or more such equations, calculations may be simplified by proceeding as follows:

For each of the equations, find the values of X, Y, XY, and X^2. In this instance, the following values are obtained:

	X	Y	XY	X^2
	- 20	1,544	- 30,880	400
	- 10	1,869	- 18,690	100
	0	2,211	0	0
	10	2,350	23,500	100
Total (Σ)	- 20	7,974	- 26,070	600

Substitute the values obtained in this table into the following normal equations:

$$\Sigma(Y) = na + b\Sigma(X)$$

$$\Sigma(XY) = a\Sigma(X) + b\Sigma(X^2)$$

Where:

Σ (X) is the total of the column headed		X
Σ (Y)	"	Y
Σ (XY)	"	XY
Σ (X^2)	"	X^2

and n is the number of equations, in this case 4.

Therefore:

$$7,974 = 4a - 20b$$

$$-26,070 = -20a + 600b$$

To solve both equations simultaneously, both sides of one equation may be multiplied by a number that would make one of its elements equal to the corresponding element in the other equation but of different sign. In this instance, multiply both sides of the first equation by 5. The next step is to add the corresponding elements of the two equations to obtain one equation with one unknown: Thus:

$$39{,}870 = 20a - 100b$$

$$\underline{-26{,}070 = -20a + 600b} \text{ by adding:}$$

$$13{,}800 = 500b$$

Therefore:

$$b = \frac{13{,}800}{500}$$

and:

$$b = 27.6$$

by substituting the value of b in either equation:

$$7{,}974 = 4a - 20\ (27.6) \qquad \text{or} \qquad -26{,}070 = -20a + 600\ (27.6)$$

$$7{,}974 = 4a - 552 \qquad\qquad -42{,}630 = -20a$$

$$\text{and} \qquad a = 2{,}131.5 \qquad\qquad a = 2{,}131.5$$

The specific equation projecting the population is, therefore, the following:

$$Y_{65} = 2{,}131.5 + 27.6X$$

The 1965 estimate of the population of Puerto Rico is, therefore, obtained by substituting 15 for X in the equation:

$$Y_{65} = 2,131.5 + 27.6\ (15)$$

and:

$$Y_{65} = 2,545.5$$

The estimated population in 1965 rounded to the first 1000 is therefore:

2,546,000. 1/

The Second Degree Polynomial

If the base period is adequately chosen, linear curves, or first degree polynomials, are usually adequate for projection purposes. In rare instances, however, the given population data fluctuate in such a way that a straight line is not deemed to fit them adequately. In such cases,

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1/ In case the number of counts included in the base period is odd, the computation could be made much simpler by making $X = 0$ to correspond to the median (or middle) count. In this case $\Sigma(X) = 0$ and the equations become:

$$\Sigma\ (Y) = na$$

$$\Sigma\ (XY) = b\ \Sigma\ (X^2)$$

hence:

$$a = \frac{\Sigma\ (Y)}{n} \qquad \text{and} \qquad b = \frac{\Sigma\ (XY)}{\Sigma\ (X^2)}$$

If the reader is unaquainted with these computations, it might be useful to work them out for the example of Swaziland where three population counts are available. Make $X = 0$ correspond to 1946. It will be found that:

$$a = 193 \qquad \text{and} \qquad b = 4$$

Therefore:

$$Y_{60} = 193 + 4\ (14) = 249,000.$$

and if these fluctuations are considered typical and likely to continue in a similar pattern into the future, the use of a second degree polynomial may be justified.

The general formula of a polynomial of the second degree is the following:

$$Y = a + bX + cX^2$$

This curve can be made to pass through three given population values and may be fitted to a larger number without necessarily passing through all of them.

To illustrate its use we shall use it in estimating the 1960 Swaziland population using the counts of 1936 (157,000), 1946 (185,000) and 1956 (237,000).

For 1936	$X = -10$	and	$Y = 157(000)$
For 1946	$X = 0$	and	$Y = 185(000)$
For 1956	$X = 10$	and	$Y = 237(000)$
For 1960	$X = 14$	and	Y_{60} is to be estimated.

By substitution into the formula of a polynomial of the second degree, the following three equations are obtained:

$$157 = a - 10b + 100c$$

$$185 = a$$

$$237 = a + 10b + 100c$$

Solve for a, b, and c:

$$\underline{a = 185}$$

and by adding:

$$\begin{array}{l} 157 = a - 10b + 100c \\ \underline{237 = a + 10b + 100c} \\ 394 = 2a + \qquad 200c \end{array}$$

by substituting the value of a, derived above:

$$394 = 2(185) + 200c$$

$$c = .12$$

and by substituting this value for c and the value of a in the earlier equation:

$$\begin{array}{l} 237 = a + 10b + 100c \\ 237 = 185 + 10b + 100(.12) \\ 10b = 40 \\ b = 4 \end{array}$$

Construct the equation of the parabolic fit and substitute the values of a, b, and c into the equation:

$$Y = a + bX + cX^2$$

$$Y = 185 + 4X + 0.12X^2$$

Hence:

$$Y_{60} = 185 + 4\,(14)^* + 0.12\,(14)^2$$

therefore:

$$Y_{60} = 265$$

and the population of Swaziland in 1960 is thus estimated at 265,000.

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* Since 1960 is 14 years following the base period 1946, X = 14.

Summary

The methods of extrapolation discussed in this chapter have been classified under three headings: Arithmetic extrapolation, simple geometric extrapolation and polynomial extrapolation. All these methods start in the same manner; namely, by a careful choice of a base period for the projection.

Arithmetic extrapolation involves two further steps: the determination of the yearly increment and the application of this increment to the projection period to obtain an estimate of total population at one or more future dates. The results of arithmetic extrapolation are exactly the same as those obtained by graphic extrapolation and those obtained by using the polynomial of the first degree.

Simple geometric extrapolation also involves two major steps after the choice of a base period is made; (1) estimating the rate of population growth (or decline) (R), or an approximation of it (r_1, r_2, or r_3), and (2) applying this growth rate in either the continuous compounding or the annual compounding formula.

Polynomial extrapolation involves the use of a polynomial function. Only the uses of first and second degree polynomial functions were introduced in this chapter because higher degree polynomials are seldom necessary for projecting populations. The first degree polynomial may represent a straight line on plain graph paper passing through the two values of population counts contained in the base period, or, in case more counts are included, a straight line that passes right in the center of their respective values. A second degree polynomial may represent a curve that passes through three different counts and, if more than three counts are included in the base period, it will represent a curve passing at the center of the values of the respective counts.

The following table shows estimates of the 1960 population of Swaziland computed through the different methods discussed in this chapter:

TABLE III

ESTIMATES OF SWAZILAND
POPULATION IN 1960 BY
ALTERNATIVE METHODS
(1946 - 1956 = base)

Method	Population 1960
Arithmetic or) Graphic or) First degree polynomial extrapolation)	258,000
Geometric Extrapolation:	
Continuous Compounding with R	262,000
" " " r_1	265,000
" " " r_2	262,000
" " " r_3	262,000
Annual Compounding with R	261,000
" " " r_1	265,000
" " " r_2	261,000
" " " r_3	262,000
Parabolic Extrapolation) second degree polynomial)	265,000

Note that: 1 - Arithmetic extrapolation gives a lower estimate than geometric extrapolation and,

2 - the only estimate which seems slightly out of balance is the one using the very rough but simple to compute approximation of R, namely r_1.

Extrapolations Using Modified Exponential, Gompertz, and Logistic Curves - Discussion [1]

The extrapolation techniques described in the preceding sections should meet most of the needs of the economic analyst working in demography. Demographers, however, have at times used different types of curves to fit and project population data. These curves are often referred to as "growth curves." Most famous among these are the modified exponential and, to a greater degree, the Gompertz and logistic curves.

These three types of curves could be viewed as being of the more complicated variety of exponential curves. The general formula for an exponential function is the following:

$$Y = ab^{x}$$

An exponential curve describes the relation in which one variable (e.g., when x = time in years) increases in arithmetic progression and the other (e.g., when Y = the size of the population P_x) increases in geometric progression. In other words, the exponential curve relates two variables (e.g., time and the size of the population); the first increasing by constant absolute increments while the corresponding values of the second change at a constant rate.

In the preceding pages the reader was, at one time, informally introduced to the exponential curve. It was used in effecting what was called "simple geometric interpolation" and "simple geometric extrapolation." The reader will recall the function applied in these two sections; namely,

$$P_n = P_o \quad e^{Rt}$$

and notice its conformity with the general formula of the exponential function.

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[1] This section is only a brief introductory discussion of major growth curves. A short selected bibliography is included on pages 244 - 245 for the reader interested in learning more about their nature and use for extrapolation.

It can easily be noted that with larger and larger values of t - i.e., for distant points in the future - the exponential curve will eventually indicate such a fantastic population size as to be absurd. It is this characteristic of the exponential curve, namely, its relentless growth, that prompted the exponents of the logistic and other asymptotic growth curves to develop their modfied versions.

For all three functions mentioned above, the curve describing population growth eventually approaches asymptotically a level prescribed by the data. In other words, the population of an area or country is believed to have, at any point in time, a limit beyond which it cannot grow. This limit is obviously set by physical and technical conditions so that it changes with them. A drastic change in these basic conditions necessitates, therefore, the construction of a new curve and, hence, a revision of previous projections not yet realized.

The equation describing the modified exponential function can be written:

$$Y = ab^x + K$$

for use in fitting population data when a is found to be negative and b to be a fraction of one. Only under these circumstances would the modified exponential curve be concave from below. Furthermore, this curve describes a trend in which the increment of change declines by a constant percentage and which eventually becomes asymptotic to the value of K.

The Gompertz and Logistic curves are given by the following two equations: [1]

The Gompertz curve: $Y = K a^{b^x}$

The Logistic curve: $\frac{1}{Y} = \frac{1}{K} + a\ b^x$

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[1] For further reference, see: Dudley J. Cowden, "Simplified Methods for Fitting Certain Types of Growth Curves", Journal of America Statistical Association, December 1947, Vol. 42, pp. 585-590.

They both contain the asymptotic feature--the upper limit to population increase but differ from the modified exponential curve in that, when they are concave from below they are S shaped rather than half-bell shaped. These curves represent a trend of growth with increasing absolute increments up to a certain point (point of inflection) beyond which these increments gradually diminish until they become negligible.

PART I

CHAPTER IV

ESTIMATING TOTAL POPULATION FOR SUB-NATIONAL REGIONS

The empirical as well as the interpolation and extrapolation methods described in the previous chapters were illustrated by examples relating to national entities. This, however, should not be construed to mean that they apply only to such entities. A little reflection on the part of the reader should make it clear that, if the data used in these different methods are available for sub-national regions, there is no reason why these methods could not be used to estimate total population for such regions.

In the present chapter two additional methods of estimating total population, the Economic and Ratio methods, will be introduced. Unlike the previous methods, these find their widest application in the framework of sub-national regions. In fact, they were originally devised specifically for use in such a framework.

The Economic Method

As the reader will soon notice, the main principles of the economic method are very similar to those expounded in connection with the Empirical Methods. The main difference is that while the Empirical Methods deal mainly with present or past population size, the Economic Method is designed to look into the future. Another difference, of course, is that the latter method is mainly devised to apply to relatively small areas exclusively.

Like the Empirical Methods, the Economic Method contains two components-- a "primary factor" and an "allowance."

To illustrate, suppose that a large industrial concern decides to move into a small community of 2,000 people. The firm itself as well as public planning agencies may be interested in determining the future size of the population in that community after this movement is completed. Using the Economic Method, one may proceed as follows:

The PRIMARY FACTOR is determined by the assumed or planned capacity of this particular concern to employ labor. Assume for the sake of this illustration that this capacity is set at 1,000 workers. If we find, after studying unemployment and underemployment conditions, that 250 workers will be secured from the community itself and 750 workers will have to in-migrate, primary factor then can be said to imply 750 persons.

Once the value of the primary factor is determined, an ALLOWANCE has to be made to take into account the additional population that the movement of the 750 workers into the new community implies. First come the dependents. Based on family characteristics of laborers in neighboring areas, the estimator may decide to place the average size of the in-migrating workers' families at 1.5 dependents to each worker. The number of dependents will, therefore, be set at 1,125 persons. Second comes the allowance that has to be made to represent additional workers in subsidiary industries and their dependents. Assume that, after investigation of the labor needs that the initial industry will create in the community (based, perhaps, on similar conditions elsewhere or even pure conjecture), it is decided that 200 additional people will in-migrate. In this case the total allowance will amount to 1,325 persons.

Following the Economic Method, therefore, total population at the projection year (P_x) is equal to the original population (P_o) plus the measure of the condition (C) plus the allowance (A). Hence:

$$P_x = P_o + C + A \text{ or, in this instance}$$

$$P_x = 2,000 + 750 + 1325 = 4075 \text{ people.}$$

The method of determining the value of the primary factor and the allowance depends primarily, of course, on the situation at hand. It is believed that they may be determined with some reliability if the following conditions are satisfied:

First, the values of the primary factor and the allowance are seriously and thoroughly investigated.

Second, the projection period is of short distance into the future, for example, not more than two or three years. Projections for longer periods will have to make a further allowance for net reproduction among the original population (perhaps at an increasing rate) and among the in-migrant.

Finally, the original population of the area must not be too large in relation to the size of in-migration. In other words, the economic method, as described here, will be less effective the smaller the value of $\frac{A + C}{P_o}$

Its results should become doubtful if $\frac{A + C}{P_o}$ is less than $\frac{1}{10}$.

Let us note before closing that the primary factor need not be an industry. It may be a housing development, a circus, or anyone of a multitude of things. The important condition is that it has an appreciable affect on the size of the original population of the area. If, for example, $P_o = O$; that is, if the area where the primary factor will operate has no original population like in the case of some housing developments; then the third condition becomes automatically satisfied.

The Ratio Method

"The Ratio Method does not of itself provide population projections; its function is solely to provide sets of proportions ('ratios') which, when multiplied by a given projection, indicate the population in the constituent parts of the area. The method specifically assumes the availability of a population projection for the over-all area. By the ratio method, however, a national projection can be successively broken down into smaller and smaller subdivisions. An advantage of the method is that the given population projection can be revised repeatedly without necessitating recomputation of the ratios." 1/

The basic data needed in applying the Ratio Method adequately are seldom existent in a developing country. This is perhaps the main reason why such a method has found no significant popularity in those parts of the world. Another reason is probably the fact that the Ratio Method is a relatively new one. 2/ However, if the trend in census taking in these countries continues at its present pace, a wide use of the Ratio Method may take place in the future. With the availability of basic data, the Ratio Method is one of the fastest techniques for forecasting populations of small areas en masse.

The basic data needed for this method are the following: (1) a reliable forecast of total population for an area which includes the smaller areas in question; and (2) a set of ratios showing the percentage of the over-all area's population residing in each of the subregions at one or, better yet, several points in the past.

Once these data are available, the past ratios for each subregion may be projected to the forecast date (perhaps, by straight line extrapolation as explained in Part I, Chapter III) and a set of ratios are thus obtained which can be used to break down the given over-all area population forecasts into its subregional components (the short method).

1/ W. Hodgkinson Jr., Estimates and Projections of the Population in Large Cities and Their Use in Urban Development Planning, United Nations, Economic and Social Council, E/CN. 9/ Conf. 2/ L.5, December 1959

2/ The publication of the U.S. Bureau of the Census in 1952 reproduced below is considered pioneering work.

Obviously, this new set of ratios may not add up to exactly 100 percent in which case "forcing" may become advisable. [1]

Frequently, it is necessary, in making small area forecasts with the ratio method, to start with a projection for a very much larger area. Here the best practice ('long' method) is to proceed in "successive stages." In other words, the only reliable forecast may be that of the population of the entire nation while the requested forecasts are of subregions that are so small that their individual shares of the total are, in some instances, negligible. Extrapolating "negligible" ratios is inadequate for a reliable projection of the small subregional populations. In such cases, the ratios of large geographic areas constituting the forecast area (i.e., the nation) are first secured and the forecasts of their respective populations are made. These forecasts will then be broken down, again by use of the ratio method, to give the forecasts of the smaller subregional areas.

To illustrate, the U.S. Bureau of the Census's State projections of 1952 used the ratio method in two stages. First, proportions (or ratios) were derived for nine geographic areas and then the ratios of the States composing these areas were computed. Following is a full reproduction of their published methodology.

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[1] Forcing a total of ratios to equal 100 percent is usually done as follows: Suppose that the total of all ratios is 105.0 percent. Also that this is arrived at by addition of the following percentage: 5, 15, 20, 30 and 35 percent. Each of these percentages should be reduced in proportion to its size so that they all add up to 100 percent. In fact, they do add up to 100 percent of 105 so that if we take the percents that each is of 105, these should represent the "forced" percentages. Hence 5 is 4.34 of 105 and the forced percentage of 5 is 4.3 etc.

U. S. DEPARTMENT OF COMMERCE
Charles Sawyer, *Secretary*

BUREAU OF THE CENSUS
Roy V. Peel, *Director*

CURRENT POPULATION REPORTS

POPULATION ESTIMATES

FOR RELEASE
January 27, 1952 Washington 25, D. C. Series P-25, No. 56

PROJECTIONS OF THE POPULATION BY STATES: 1955 AND 1960*

This report presents projections of the population of each region, geographic division, and State, for 1955 and 1960, taking into account the 1950 Census totals for these areas. The projections are designed to represent the civilian population of each area plus members of the armed forces who resided in the area at the time of their entry into the armed forces. This type of population cannot be enumerated easily or reliably in a census, but is the type for which the most realistic assumptions can be made as to future change and for which the most useful projections can be provided. Users of these projections can then develop projections of the resident population of each area by making whatever assumptions as to future military changes they consider appropriate.

These projections are being published at this time, even though they are subject to relatively large errors, because the demand for figures of this kind has been considerable. It is believed that they are reliable enough to serve many important purposes and that persons working in the fields of public planning and market analysis will find them useful.

SOURCES, METHODS, AND ASSUMPTIONS

General method.--A "ratio" method was selected after consideration had been given, from the point of view of validity and cost, to several possible methods of projecting State populations. Briefly, the ratio method consists of (1) extrapolating the ratio of (a) the population of the area for which a projection is desired to (b) the population of a larger area which includes the first area and for which acceptable population projections are already available; and (2) applying the extrapolated ratios to the population projections for the larger area to obtain projections for the smaller area. In preparing the projections for geographic divisions shown in this report, the ratio of the division total to the United States total was extrapolated and the extrapolated ratio was applied to projections of United States total population; in preparing projections for States, the ratio of the State total to the appropriate division total was extrapolated and the extrapolated ratio was applied to the projections of the division total. Regional projections were obtained by combining the appropriate divisional figures. The specific assumptions and procedures used are discussed below.

Specific assumptions and procedures.--First, the ratio of the population of each division to the total population of the United States and the ratio of the population of each State to the total population of its division were computed for each decennial census year from the earliest census to 1950. The ratios for 1920 to 1950 are given in table 1. On the basis of these data, the divisions and States were next divided into the following three groups:

Group 1. Those areas for which the ratios show a consistent direction of change from 1920 to 1950.

Group 2. Those areas for which the direction of change in the ratios from 1940 to 1950 was the same as from 1930 to 1940 but not as the change from 1920 to 1930.

Group 3. Those areas for which the direction of change in the ratios from 1940 to 1950 was different from that for 1930 to 1940.

---

* Prepared by Helen L. White, formerly statistician in the Estimates and Forecasts Unit of the Population and Housing Division, and Jacob S. Siegel, Chief of the Estimates and Forecasts Unit.

The following assumptions were then made for each group with respect to the initial annual rate of change in the ratio:[1]

Group 1. The rate of change in the ratio was the same as the average annual rate of change in the ratio for 1920-50, 1930-50, or 1940-50, whichever was the least in absolute value (closest to zero).

Group 2. The rate was the same as the average annual rate for 1930-50 or 1940-50, whichever was the lesser.

Group 3. The rate was one-half of the average annual rate for 1940-50.

The rates of change so determined were assumed in the computations to apply to the period, July 1, 1950, to June 30, 1951. They are shown in table 1. It was also assumed that the annual rates of change would decrease linearly to zero in 50 years, that is, by the year 2000-2001. Accordingly, values for the annual rates of change assumed to apply to each year, 1951-52 to 1959-60, were obtained by linear interpolation of the initial and terminal values.

Preliminary values of the ratios for July 1, 1955, and July 1, 1960, were then computed by multiplying the ratios for July 1, 1950, serially by one plus the projected annual rates of change for the appropriate years.[2] The preliminary projected ratios for geographic divisions, and for the States within each division, for 1955 and for 1960, were then adjusted to sum to exactly 100 percent.[3] The adjusted ratios are shown in table 2. Finally, projections of the population of each division for July 1, 1955 and 1960, were obtained by applying the adjusted ratios for divisions to projections of the total population of the United States (including armed forces overseas) for 1955 and 1960, and projections of the population of each State were then obtained by applying the adjusted State ratios to the projected totals for divisions for these years. By using "low," "medium," and "high" projections of the total United States population at future dates, three series of projections for States and divisions were developed for July 1, 1955 and 1960. The resulting projections are shown in table 3.

Definition of population.--Such factors as the change in the number of armed forces personnel stationed in each State and overseas, and the movement into and out of the armed forces, can generally be regarded as disturbing the "normal" trend both of the ratios and of the population figures. The preparation of projections of the civilian population of an area and of the total population resident in the area (the civilian population plus members of the armed forces stationed there) involves making projections of these military changes. It was decided, in the present instance, therefore, to work with, and develop projections for, a more regular and presumably more predictable population, comprising the civilian population and those members of the armed forces who resided in the area at the time of their entry into the armed forces. The preparation of this series does not involve making separate projections of military changes, as do the two other types of estimates mentioned. (For all areas combined, this type of figure represents the total population of the United States including armed forces overseas.) Users of these projections who desire projections of the total population residing in each area and of the civilian population can develop them from the projections presented here by making whatever assumptions regarding future military strength and distribution they consider appropriate, perhaps on the basis of the differences between the corresponding types of estimates for some recent date.

The census data for April 1, 1950, and the population estimates for July 1, 1950, used in preparing these projections, were adjusted in accordance with this definition before the computation of ratios. (Census data for 1940 and prior years were not adjusted on this basis because of the small number of military personnel involved.) Correspondingly, the projected ratios for divisions were applied to projections of the total population of the United States including armed forces overseas to obtain estimates of the population of each division as defined above.

Basic data.--These population projections are based on data on the population of States from the various decennial censuses, the figures for 1950 being given in 1950 Census of Population, Series PC-9, No. 1; estimates of the population of the States for July 1, 1950, published in Current Population Reports, Series P-25, No. 50; projections of the population of the United States for 1955 and 1960, published in Current Population Reports, Series P-25, No. 43; and data on the size and distribution of the armed forces for 1950 provided by the Department of Defense.

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[1] The usual formula for the average annual rate of change in a series, say a series of proportions, is $\sqrt[t]{\frac{P_1}{P_0}} - 1$, where P_1 represents the proportion at the end of the period, P_0 the proportion at the beginning of the period, and t represents the number of years in the period. In order to simplify the procedure, the average annual rate of change was approximated by use of the formula $\frac{2(P_1 - P_0)}{t(P_1 + P_0)}$, which gives a satisfactory approximation when $\frac{P_1}{P_0}$ falls between 0.5 and 1.5 for the 10-, 20-, and 30-year time spans considered here.

[2] That is, each ratio for 1950 was multiplied by one plus the rate of change assumed for 1950-51, the product was multiplied by one plus the rate assumed for 1951-52, and so on. It may be noted that, in computing the average annual rate of change in the various ratios for periods ending in 1950, the value of the ratio on April 1, 1950, was used; however, in extrapolating the ratios, the initial annual rate of change was applied to an estimate of the ratio for July 1, 1950.

[3] In the case of a few of the areas, this adjustment had the effect of distorting slightly the trend in the ratios originally projected, but no further adjustment was made to eliminate this distortion.

LIMITATIONS OF THE PROJECTIONS

A definite statement as to the reliability of the projections for States, divisions, and regions presented in this report cannot, of course, be given. Suggestive comments can be made, however.

It should be recognized first of all that these projections represent the results of the use of a certain method and a certain set of assumptions; they must be interpreted, therefore, in the light of this method and these assumptions. Other methods and other reasonable assumptions could have been used which would have produced somewhat different results. The method selected appeared to offer the best results for the limited funds available for preparing the projections.

Since the projections are based on the projections of the total population of the United States published in Current Population Reports, P-25, No. 43, they are affected by the limitations of these projections, as described on page 7 of that report. Since, in addition, the projected ratios for each division and State are subject to some error, the projections for these areas are, on the average, subject to greater error than the national projections.

Within the framework of the present assumptions, the range set by the high and low series gives an indication of the range of possible error--probably a minimal one. The high projections for 1960 exceed the low projections by 11.5 percent. If three series of ratios, instead of one, had been used in developing the present series of population projections--a design which seems reasonable and which may be preferred by some--the resulting figures would possibly have had a considerably broader range--too broad perhaps to make the figures practically useful.

To date, no adequate test of the relative validity of various methods of projecting the population of geographic areas within the United States has been made.[4] A preliminary test of this kind is now being made at the Bureau of the Census, and a full description of the design of the test and of the results will shortly be published. In general, the test involves projecting the population of each State from 1930 to 1940 and 1950 by various methods and comparing the results with the 1940 and 1950 Census results. Comparisons were made between projections prepared by such methods as the following: the cohort-survival method; geometric extrapolation; arithmetic extrapolation; and several variations of the ratio method, including the variation employed in this report.[5] It is believed, on the basis of the preliminary results of this test, that, in general, the projections for the States with a relatively large population in 1950 and with relatively little net migration in the recent past are subject to a smaller percent error than the projections for States with a relatively small population and relatively large net migration. Also, the projections for 1955 are, on the average, likely to be considerably more accurate than the projections for 1960. The maximum percentage error shown in the test for any area, for the 10-year projections, when the particular variation of the ratio method employed in this report was used, was 24 percent (District of Columbia); if the District of Columbia is excluded, the maximum was 15 percent.

RELATED REPORTS

Related estimates.--Reference has already been made to the 1950 Census data for States and to the estimates of State population for July 1, 1950. Estimates of the total population of States for July 1 of each year, 1940 to 1949, have been published in Current Population Reports, Series P-25, No. 47. The projections and current figures given in the present report, however, should not be used in conjunction with these data or other estimates for dates after April 1, 1940, published in other reports in the P-25 series, unless differences in the treatment of the armed forces are taken into account. The figures for the total population of States published in Series P-25, Nos. 47 and 50, and in Series PC-9, No. 1, relate to the civilian population plus armed forces stationed in the area; as indicated earlier, the figures in this report relate to the civilian population plus those members of the armed forces who resided in the area at the time of their entry into military service.

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[4] A test of limited scope, relating solely to the validity of the ratio method in predicting the population of selected large cities, was recently conducted by Robert C. Schmitt and Albert H. Crosetti and is described in "Accuracy of the Ratio Method for Forcasting City Population," Land Economics, Vol. XXVII, No. 4, November 1951, pp. 346-348.

[5] The cohort-survival method involves projecting the population as enumerated at the last census or as estimated for a current date, by age and sex, to a future date by use of projected birth rates, death rates, and migration. This method is described in detail in Current Population Reports, Series P-25, No. 43. Geometric extrapolation and arithmetic extrapolation involve, respectively, the assumption of a continuation of the average annual rate and average annual amount of increase in the population as observed in some recent past period.

Related projections.--The latest available projections of the population of the United States as a whole are those published in Current Population Reports, Series P-25, No. 43. This report presents three series of annual figures to 1960, classified by age and sex. These figures were developed on the basis of current estimates for July 1, 1949; they do not take the 1950 Census and information on subsequent population changes into account. On the basis of these recent indications, it is anticipated that as of January 1, 1952, the current estimates of the total population of the United States will fall about midway between the medium and high projections implied for that date. Although revision of these projections may appear desirable on this account, it was not deemed necessary or feasible to carry out a revision at this time for the purpose of preparing these State projections.

Projections of the population of geographic divisions for 1955, 1960, and 1975 were recently prepared by Margaret J. Hagood and Jacob S. Siegel.[6] The general method used in preparing those projections is the same as that used in the present report. Because of differences in the basic data used, the specific assumptions selected, and the details of computation, however, the corresponding projections--those for divisions in 1955 and 1960--are not in exact agreement. The maximum difference, that for the East South Central division in 1960, is about 4 percent; but seven out of the nine divisions show differences under 2 percent. It was considered desirable in preparing the present series to make use of the Census Bureau's State estimates for July 1, 1950, which became available after the earlier projections were prepared, as well as to use an approach, in setting up the assumptions for projecting the population ratios, which permitted more precise formulation. The article cited presents, in addition to the basic series mentioned, projections of the age-sex distribution of the population of the four major geographic regions in 1960.

PROJECTIONS FOR OTHER AREAS AND DATES

Frequently, projections of the population of areas other than States or groups of States, or for earlier and later dates than given here, are needed. The method and assumptions used in preparing the present projections can generally be extended to prepare these additional kinds of figures. If the past rate of growth of the population of an area has paralleled, even roughly, the rate of growth in the State, it may be sufficient to use (without extrapolation) the proportion of the total population of the State in the area, as shown by the 1950 Census or more recent data, in conjunction with the projected State total.

Since it appears that the reliability of projections generally decreases as the size of the area decreases and as the estimate date extends further into the future, projections for more than a decade hence of relatively small populations, say of several hundred thousand or less, are probably subject to considerable error. The error may frequently be large enough to render such projections inadequate for most of the uses to which they may be put. Futhermore, it is recommended that projections for more than five or so years into the future not be attempted for rapidly growing areas with populations of fewer than 50,000 persons. (These limits are somewhat arbitrary, but they may serve as convenient guides until limits are developed by empirical test.) In projecting the population of a geographic area within the United States, particularly a rather small area, direct consideration should be given, insofar as possible, to the economic, industrial, and social structure of the area. For some very small areas, the expansion or contraction of a single industry or even a single firm may be the determining factor in the course of population changes. Any projections should be carefully checked for consistency with past trends and for reasonableness in the light of expected future developments, and consideration should be given to the preparation of several projections using different methods.

If there is interest in projections of the age-sex distribution of the population of States, the ratio method applied here to project totals or the method described in the Hagood-Siegel article cited above for projecting the age-sex distribution of the population of regions can be adapted to that purpose.

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[6] Margaret Jarman Hagood and Jacob S. Siegel, "Projections of the Regional Distribution of the Population of the United States to 1975," Agricultural Economics Research, Vol. III, No. 2, April 1951, pp. 41-52. This article is pertinent also for its brief discussion of the history of projections for geographic areas within the United States, its detailed description of the ratio method, its discussion of other methods, and its selected bibliography bearing on these subjects.

Table 1.--COMPUTATION OF THE RATE OF CHANGE IN THE PROPORTION OF THE POPULATION IN GEOGRAPHIC DIVISIONS AND STATES ASSUMED FOR THE INITIAL YEAR OF THE PROJECTION PERIOD

(The sum of the percentages in each distribution shown may not equal 100.00 because of rounding. Percentages for 1920 to 1940 are based directly on data from the decennial censuses; those for 1950 are based on data from the 1950 Census adjusted to include members of the armed forces residing in the area at the time of entry into the armed forces and to exclude all other members of the armed forces stationed in the area in April 1950. See pages 1 and 2 of text for further explanation)

Division and State	Percentage distribution of population				Group	Period on which rate is based	Rate of change in initial projection year (percent)
	1920	1930	1940	1950			
United States	100.00	100.00	100.00	100.00	...	...	...
New England	7.00	6.65	6.41	6.20	1	1930-50	-0.35
Middle Atlantic	21.06	21.39	20.92	20.08	2	1930-50	-0.31
East North Central	20.32	20.60	20.22	20.24	3	1940-50	0.00
West North Central	11.87	10.83	10.27	9.37	1	1930-50	-0.72
South Atlantic	13.23	12.86	13.54	13.96	2	1940-50	+0.31
East South Central	8.41	8.05	8.19	7.63	3	1940-50	-0.35
West South Central	9.69	9.92	9.92	9.63	3	1940-50	-0.15
Mountain	3.16	3.02	3.15	3.36	2	1930-50	+0.54
Pacific	5.27	6.67	7.39	9.52	1	1930-50	+1.76
NEW ENGLAND	100.00	100.00	100.00	100.00	...	...	...
Maine	10.38	9.76	10.04	9.86	3	1940-50	-0.09
New Hampshire	5.99	5.70	5.83	5.74	3	1940-50	-0.07
Vermont	4.76	4.40	4.26	4.09	1	1920-50	-0.50
Massachusetts	52.05	52.04	51.16	50.38	1	1920-50	-0.11
Rhode Island	8.17	8.42	8.45	8.36	3	1940-50	-0.06
Connecticut	18.65	19.68	20.26	21.57	1	1930-50	+0.46
MIDDLE ATLANTIC	100.00	100.00	100.00	100.00	...	...	...
New York	46.65	47.93	48.94	49.13	1	1940-50	+0.04
New Jersey	14.18	15.39	15.11	15.95	3	1940-50	+0.27
Pennsylvania	39.17	36.68	35.95	34.92	1	1930-50	-0.25
EAST NORTH CENTRAL	100.00	100.00	100.00	100.00	...	...	...
Ohio	26.82	26.27	25.94	26.17	3	1940-50	+0.04
Indiana	13.65	12.80	12.87	12.97	2	1930-50	+0.06
Illinois	30.20	30.16	29.66	28.57	1	1920-50	-0.18
Michigan	17.08	19.14	19.74	20.96	1	1930-50	+0.45
Wisconsin	12.26	11.62	11.78	11.33	3	1940-50	-0.20
WEST NORTH CENTRAL	100.00	100.00	100.00	100.00	...	...	...
Minnesota	19.03	19.28	20.66	21.23	1	1940-50	+0.27
Iowa	19.16	18.58	18.78	18.66	3	1940-50	-0.03
Missouri	27.14	27.29	28.00	28.16	1	1940-50	+0.06
North Dakota	5.16	5.12	4.75	4.41	1	1920-50	-0.51
South Dakota	5.07	5.21	4.76	4.63	2	1940-50	-0.27
Nebraska	10.33	10.36	9.73	9.43	2	1940-50	-0.32
Kansas	14.10	14.15	13.32	13.47	3	1940-50	+0.06
SOUTH ATLANTIC	100.00	100.00	100.00	100.00	...	...	...
Delaware	1.59	1.51	1.50	1.52	3	1940-50	+0.08
Maryland	10.36	10.33	10.22	11.03	3	1940-50	+0.38
District of Columbia	3.13	3.08	3.72	3.69	3	1940-50	-0.05
Virginia	16.51	15.33	15.02	15.45	3	1940-50	+0.14
West Virginia	10.46	10.95	10.67	9.63	2	1930-50	-0.64
North Carolina	18.29	20.07	20.04	19.22	2	1930-50	-0.22
South Carolina	12.03	11.01	10.66	10.04	1	1930-50	-0.46
Georgia	20.70	18.42	17.53	16.32	1	1930-50	-0.60
Florida	6.92	9.30	10.65	13.10	1	1930-50	+1.70
EAST SOUTH CENTRAL	100.00	100.00	100.00	100.00	...	...	...
Kentucky	27.17	26.44	26.40	25.53	1	1930-50	-0.18
Tennessee	26.29	26.46	27.05	28.73	1	1920-50	+0.29
Alabama	26.40	26.76	26.28	26.78	3	1940-50	+0.09
Mississippi	20.13	20.33	20.26	18.96	2	1930-50	-0.35
WEST SOUTH CENTRAL	100.00	100.00	100.00	100.00	...	...	...
Arkansas	17.11	15.23	14.92	13.28	1	1930-50	-0.69
Louisiana	17.56	17.26	18.09	18.54	2	1940-50	+0.24
Oklahoma	19.80	19.68	17.88	15.46	1	1920-50	-0.81
Texas	45.53	47.83	49.10	52.72	1	1920-50	+0.48
MOUNTAIN	100.00	100.00	100.00	100.00	...	...	...
Montana	16.45	14.52	13.48	11.73	1	1930-50	-1.06
Idaho	12.95	12.02	12.65	11.71	3	1940-50	-0.38
Wyoming	5.83	6.09	6.04	5.62	2	1930-50	-0.41
Colorado	28.17	27.98	27.07	26.06	1	1920-50	-0.26
New Mexico	10.80	11.44	12.81	13.29	1	1940-50	+0.37
Arizona	10.02	11.77	12.03	14.78	1	1930-50	+1.14
Utah	13.47	13.72	13.26	13.67	3	1940-50	+0.15
Nevada	2.32	2.46	2.66	3.13	1	1920-50	+0.98
PACIFIC	100.00	100.00	100.00	100.00	...	...	...
Washington	24.37	19.08	17.84	16.27	1	1930-50	-0.79
Oregon	14.07	11.64	11.20	10.65	1	1930-50	-0.44
California	61.56	69.28	70.97	73.08	1	1930-50	+0.27

Table 2.--PERCENTAGE DISTRIBUTION OF THE POPULATION OF THE UNITED STATES BY GEOGRAPHIC DIVISIONS AND OF THE POPULATION OF DIVISIONS BY STATES, FOR 1950, AND PROJECTED PERCENTAGES FOR 1955 AND 1960

(The sum of the percentages in each distribution shown may not equal 100.00 because of rounding. Figures relate to July 1. The estimates shown for 1950 are based on estimates of State population published in Current Population Reports, Series P-25, No. 50, adjusted to include members of the armed forces residing in the area at the time of entry into the armed forces and to exclude all other members of the armed forces stationed in the area on the estimate date. The projected percentages for 1955 and 1960 relate to a similar population)

Division and State	Estimate, 1950	Projection	
		1955	1960
United States	100.00	100.00	100.00
New England	6.19	6.08	5.97
Middle Atlantic	20.12	19.78	19.46
East North Central	20.23	20.19	20.14
West North Central	9.40	9.06	8.76
South Atlantic	13.96	14.14	14.29
East South Central	7.62	7.47	7.34
West South Central	9.61	9.52	9.43
Mountain	3.37	3.45	3.53
Pacific	9.50	10.31	11.08
NEW ENGLAND	100.00	100.00	100.00
Maine	9.90	9.85	9.81
New Hampshire	5.76	5.74	5.72
Vermont	4.10	4.00	3.91
Massachusetts	50.34	50.06	49.81
Rhode Island	8.34	8.31	8.29
Connecticut	21.57	22.04	22.46
MIDDLE ATLANTIC	100.00	100.00	100.00
New York	49.15	49.29	49.42
New Jersey	15.96	16.19	16.40
Pennsylvania	34.89	34.52	34.19
EAST NORTH CENTRAL	100.00	100.00	100.00
Ohio	26.12	26.13	26.13
Indiana	12.98	13.00	13.01
Illinois	28.58	28.28	28.00
Michigan	20.97	21.39	21.77
Wisconsin	11.34	11.21	11.10
WEST NORTH CENTRAL	100.00	100.00	100.00
Minnesota	21.23	21.50	21.74
Iowa	18.66	18.62	18.58
Missouri	28.12	28.18	28.24
North Dakota	4.43	4.32	4.22
South Dakota	4.65	4.58	4.53
Nebraska	9.44	9.29	9.16
Kansas	13.48	13.51	13.54
SOUTH ATLANTIC	100.00	100.00	100.00
Delaware	1.52	1.52	1.52
Maryland	11.04	11.21	11.36
District of Columbia	3.62	3.60	3.58
Virginia	15.42	15.49	15.54
West Virginia	9.63	9.31	9.03
North Carolina	19.26	19.02	18.78
South Carolina	10.05	9.81	9.58
Georgia	16.33	15.83	15.37
Florida	13.14	14.21	15.23
EAST SOUTH CENTRAL	100.00	100.00	100.00
Kentucky	25.56	25.34	25.15
Tennessee	28.73	29.14	29.51
Alabama	26.75	26.87	26.97
Mississippi	18.96	18.65	18.37
WEST SOUTH CENTRAL	100.00	100.00	100.00
Arkansas	13.28	12.80	12.37
Louisiana	18.55	18.69	18.80
Oklahoma	15.45	14.79	14.21
Texas	52.72	53.73	54.61
MOUNTAIN	100.00	100.00	100.00
Montana	11.77	11.18	10.66
Idaho	11.70	11.48	11.27
Wyoming	5.61	5.50	5.39
Colorado	26.08	25.74	25.42
New Mexico	13.38	13.61	13.80
Arizona	14.73	15.54	16.29
Utah	13.63	13.72	13.78
Nevada	3.10	3.24	3.37
PACIFIC	100.00	100.00	100.00
Washington	16.29	15.66	15.12
Oregon	10.66	10.43	10.22
California	73.05	73.91	74.67

Table 3.--PROJECTIONS OF THE POPULATION OF REGIONS, DIVISIONS, AND STATES, FOR 1955 AND 1960, WITH CURRENT FIGURES FOR 1950

(Totals shown may differ from the sum of parts shown because of rounding. Figures relate to July 1 and represent the civilian population of each area plus members of the armed forces who resided in the area at the time of their entry into the armed forces. The estimates shown for 1950 are based on the estimates of State population published in Current Population Reports, Series P-25, No. 50, adjusted to represent the type of population defined above)

Region, division, and State	Estimate, 1950	Low series		Medium series		High series	
		1955	1960	1955	1960	1955	1960
United States	[1]151,672,000	158,176,000	161,679,000	161,748,000	169,371,000	166,179,000	180,276,000
REGIONS:							
Northeastern States	39,910,000	40,893,000	41,109,000	41,817,000	43,064,000	42,962,000	45,837,000
North Central States	44,938,000	46,268,000	46,724,000	47,313,000	48,947,000	48,609,000	52,099,000
The South	47,296,000	49,243,000	50,226,000	50,355,000	52,615,000	51,734,000	56,003,000
The West	19,528,000	21,772,000	23,620,000	22,264,000	24,744,000	22,873,000	26,337,000
NORTHEASTERN STATES:							
New England	9,393,000	9,611,000	9,649,000	9,828,000	10,108,000	10,097,000	10,759,000
Middle Atlantic	30,517,000	31,282,000	31,460,000	31,989,000	32,957,000	32,865,000	35,079,000
NORTH CENTRAL STATES:							
East North Central	30,686,000	31,942,000	32,567,000	32,663,000	34,117,000	33,558,000	36,313,000
West North Central	14,252,000	14,326,000	14,157,000	14,650,000	14,831,000	15,051,000	15,785,000
THE SOUTH:							
South Atlantic	21,171,000	22,363,000	23,102,000	22,868,000	24,201,000	23,494,000	25,760,000
East South Central	11,552,000	11,821,000	11,870,000	12,088,000	12,435,000	12,419,000	13,236,000
West South Central	14,573,000	15,059,000	15,253,000	15,399,000	15,979,000	15,821,000	17,008,000
THE WEST:							
Mountain	5,117,000	5,465,000	5,701,000	5,588,000	5,972,000	5,741,000	6,357,000
Pacific	14,411,000	16,307,000	17,919,000	16,675,000	18,772,000	17,132,000	19,981,000

[1] This figure differs slightly from the corresponding figure for the same date published in Current Population Reports, Series P-25, No. 55, which includes among the United States armed forces overseas those whose pre-service residence was in a United States Territory or possession.

Table 3.—PROJECTIONS OF THE POPULATION OF REGIONS, DIVISIONS, AND STATES, FOR 1955 AND 1960, WITH CURRENT FIGURES FOR 1950—Con.

(Totals shown may differ from the sum of parts shown because of rounding. Figures relate to July 1 and represent the civilian population of each area plus members of the armed forces who resided in the area at the time of their entry into the armed forces. The estimates shown for 1950 are based on the estimates of State population published in Current Population Reports, Series P-25, No. 50, adjusted to represent the type of population defined above)

Region, division, and State	Estimate, 1950	Low series		Medium series		High series	
		1955	1960	1955	1960	1955	1960
NEW ENGLAND:							
Maine	930,000	947,000	946,000	968,000	991,000	995,000	1,055,000
New Hampshire	541,000	552,000	552,000	564,000	579,000	580,000	616,000
Vermont	385,000	384,000	377,000	393,000	395,000	404,000	421,000
Massachusetts	4,728,000	4,811,000	4,806,000	4,920,000	5,035,000	5,055,000	5,359,000
Rhode Island	783,000	799,000	800,000	817,000	838,000	839,000	892,000
Connecticut	2,026,000	2,118,000	2,167,000	2,166,000	2,270,000	2,225,000	2,417,000
MIDDLE ATLANTIC:							
New York	14,999,000	15,420,000	15,546,000	15,768,000	16,286,000	16,200,000	17,335,000
New Jersey	4,872,000	5,065,000	5,159,000	5,180,000	5,404,000	5,321,000	5,752,000
Pennsylvania	10,646,000	10,798,000	10,755,000	11,041,000	11,266,000	11,344,000	11,992,000
EAST NORTH CENTRAL:							
Ohio	8,016,000	8,345,000	8,508,000	8,534,000	8,913,000	8,767,000	9,487,000
Indiana	3,983,000	4,151,000	4,236,000	4,245,000	4,438,000	4,361,000	4,723,000
Illinois	8,771,000	9,032,000	9,119,000	9,236,000	9,553,000	9,489,000	10,168,000
Michigan	6,435,000	6,832,000	7,089,000	6,986,000	7,427,000	7,178,000	7,905,000
Wisconsin	3,481,000	3,582,000	3,614,000	3,663,000	3,786,000	3,763,000	4,030,000
WEST NORTH CENTRAL:							
Minnesota	3,025,000	3,080,000	3,078,000	3,149,000	3,224,000	3,236,000	3,432,000
Iowa	2,659,000	2,667,000	2,631,000	2,728,000	2,756,000	2,802,000	2,934,000
Missouri	4,007,000	4,037,000	3,998,000	4,129,000	4,188,000	4,242,000	4,457,000
North Dakota	631,000	619,000	598,000	633,000	626,000	650,000	666,000
South Dakota	662,000	657,000	641,000	671,000	671,000	690,000	715,000
Nebraska	1,345,000	1,331,000	1,296,000	1,361,000	1,358,000	1,398,000	1,445,000
Kansas	1,921,000	1,936,000	1,916,000	1,979,000	2,007,000	2,034,000	2,137,000
SOUTH ATLANTIC:							
Delaware	321,000	340,000	351,000	347,000	368,000	357,000	391,000
Maryland	2,336,000	2,508,000	2,625,000	2,564,000	2,750,000	2,635,000	2,927,000
District of Columbia	766,000	805,000	827,000	823,000	867,000	846,000	923,000
Virginia	3,265,000	3,465,000	3,589,000	3,543,000	3,760,000	3,640,000	4,002,000
West Virginia	2,038,000	2,082,000	2,086,000	2,130,000	2,185,000	2,188,000	2,326,000
North Carolina	4,078,000	4,253,000	4,339,000	4,349,000	4,545,000	4,468,000	4,838,000
South Carolina	2,128,000	2,193,000	2,214,000	2,243,000	2,319,000	2,304,000	2,468,000
Georgia	3,458,000	3,540,000	3,552,000	3,620,000	3,721,000	3,719,000	3,960,000
Florida	2,781,000	3,177,000	3,518,000	3,249,000	3,686,000	3,338,000	3,923,000
EAST SOUTH CENTRAL:							
Kentucky	2,952,000	2,996,000	2,985,000	3,064,000	3,127,000	3,147,000	3,329,000
Tennessee	3,319,000	3,445,000	3,502,000	3,522,000	3,669,000	3,619,000	3,905,000
Alabama	3,090,000	3,176,000	3,202,000	3,248,000	3,354,000	3,337,000	3,570,000
Mississippi	2,190,000	2,205,000	2,181,000	2,254,000	2,284,000	2,316,000	2,432,000
WEST SOUTH CENTRAL:							
Arkansas	1,936,000	1,927,000	1,887,000	1,970,000	1,976,000	2,024,000	2,104,000
Louisiana	2,704,000	2,815,000	2,868,000	2,878,000	3,005,000	2,957,000	3,198,000
Oklahoma	2,251,000	2,227,000	2,168,000	2,277,000	2,271,000	2,340,000	2,418,000
Texas	7,683,000	8,090,000	8,330,000	8,273,000	8,726,000	8,500,000	9,288,000
MOUNTAIN:							
Montana	602,000	611,000	608,000	625,000	637,000	642,000	678,000
Idaho	599,000	627,000	643,000	641,000	673,000	659,000	717,000
Wyoming	287,000	300,000	307,000	307,000	322,000	316,000	343,000
Colorado	1,335,000	1,407,000	1,449,000	1,438,000	1,518,000	1,478,000	1,616,000
New Mexico	685,000	744,000	787,000	760,000	824,000	781,000	877,000
Arizona	754,000	849,000	929,000	869,000	973,000	892,000	1,036,000
Utah	697,000	750,000	786,000	766,000	823,000	787,000	876,000
Nevada	158,000	177,000	192,000	181,000	202,000	186,000	215,000
PACIFIC:							
Washington	2,348,000	2,554,000	2,709,000	2,612,000	2,838,000	2,684,000	3,020,000
Oregon	1,536,000	1,700,000	1,831,000	1,739,000	1,918,000	1,786,000	2,041,000
California	10,527,000	12,053,000	13,380,000	12,325,000	14,017,000	12,663,000	14,919,000

PART II

ESTIMATING AND PROJECTING COMPONENTS OF POPULATION CHANGE

CHAPTER I

THE COMPONENTS OF POPULATION CHANGE

Methods devised mainly for use in estimating and projecting total population were introduced in Part I. In Part II, however, methods will be investigated which are designed for estimating and projecting the components of population change. Naturally, if the different components of population change are estimated, their total may serve as an estimate of total change. The methods we shall discuss presently, therefore, furnish, so to speak, more information about population growth than those discussed in Part I. The reason why the methods of Part I are, nevertheless, still useful and essential to the demographer, is that they are much less tedious to apply and, with developing countries in mind, require much less basic data than the component method. Furthermore, even when data and time are plentiful, the projections of total population may represent checks on the results of the component method.

Total population may be broken down into a variety of components. The respective numbers of males and females in a population represent its sex components while the numbers of persons in each age group constitute its age components. If a population is classified by age groups for each of the two sexes, it is said to be broken down into its age-sex components. Similarly, a population may be broken down into its ethnic components if it contains different ethnic groups; into its linguistic components if more than one "first language" exists among the different groups; into its rural-urban components; its religious components; and so on.

Population growth implies a change in every set of components. If a population increases, the number of males and/or females will have to increase. Even though, in some instances, one item in a given set of components may remain fairly constant in spite of a significant change in total population, the other item will necessarily have to show a change equivalent to the change in total population.

Looking at population growth not from the different census classifications but from the origins of change, another important set of components is found, namely, natural increase and migration. For lack of a better expression, these will be called the growth components of population change.

The component methods that will be presented in the next three chapters involve the estimation and projection of the growth components of population change classified by age and sex. In other words, natural increase and migration will be estimated and projected for each age-sex group in the population. Obviously, the growth components may be broken down into one or more of the different sets of components mentioned above, such as ethnic and religious components, but the general technique remains unaltered irrespective of the breakdown so that our illustrations will not, in any case, lose their usefulness.

In his pioneering work in this field of manpower techniques, A. J. Jaffe states that, "There are three types of population estimates; these differ from one another with reference to the time period involved. These three are:

(1) inter-censal estimates

(2) post-censal estimates

(3) future estimates

"Inter-censal estimates are those made for the years (or other time periods) between any two consecutive censuses, and are based on data from both censuses.

"Post-censal estimates are those made for the period following a census up to the present moment of time, and are based on data from the last census.

"Future estimates are those made for any period of time after the present moment." [1]

The techniques used in making "post-censal estimates" differ from those of "future estimates" in that they involve the use of current data, mainly school enrollment of children. The assumption underlying the use of school enrollment implies either that no significant change in the proportion of children attending school takes place over the projection period, or that this change is known and measurable. This assumption is too strong to make in most developing countries. We shall, therefore, use the techniques of "future estimates" to apply to any period following the last census.

In the following chapter, estimating the growth components (by age and sex) between two censuses (inter-censal estimates) will be discussed; and in Chapter III, the use of the component method for population projection into the future (future estimates) is introduced. In Chapter IV, a short-cut method for making population projections by age and sex will be explained. Finally, the last chapter in Part II will be concerned with techniques of estimating and projecting the number of persons reaching a given age annually.

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[1] A. J. Jaffe, Handbook of Statistical Methods for Demographers, third printing, U.S. Department of Commerce, Bureau of the Census, Washington, 1960, p-211.

PART II
CHAPTER II

MEASURING THE GROWTH COMPONENTS OF POPULATION CHANGE BETWEEN TWO CENSUSES

The component method may be summarized by the following equation:

$$\Delta P = N_i + M$$

Where, ΔP is the change in the size of the population between the two censuses, i.e., the latter count minus the earlier count, N_i is natural increase during the period, i.e., births minus deaths, and M is net migration or the difference between in-migration and out-migration.

When dealing with a period of time between two censuses, as we intend to do presently, ΔP is readily available. An estimate of either net migration or natural increase is sufficient to solve the equation. In other words, when ΔP is known, an estimate of natural increase results in an estimate of net migration (by subtracting the estimated N_i from ΔP) while an estimate of net migration, in turn, implies an estimate of natural increase (by subtracting the estimated M from ΔP). Therefore, when total population change (ΔP) is known, the estimator using the component method may treat any one of the two growth components as a "residue" by estimating the other.

Which component is to be treated as a residue depends on the availability of data. In the case of national entities, records showing the number of persons entering and leaving the country--perhaps by age and sex, or any other required breakdown--may be readily available in the records of offices of tourism, immigration or border police. In such a case, natural increase may be treated as residue while net migration is directly obtained from the records. However, is such records do not exist, or are unreliable or not easily adjustable, it may become simpler, if not more adequate, to estimate natural increase and treat net migration as a residue.

To estimate natural increase, separate estimates of births and deaths are generally required. If records of births and deaths--again by the feasible breakdown (e.g. sex)--are available, they will most probably contain inaccuracies and must be adjusted. 1/ Once adjusted, their difference may be taken as an estimate of N_i. This estimate may then be subtracted from total population change (ΔP) and the residue taken as an estimate of net migration.

In brief, when population change over a period of time is known, one of the two growth components may be secured or estimated and the other treated as a residue. If estimates of both components are available, the more reliable of the two should be used and the difference between the residue and the available estimate of the other component taken as an approximate measure of the bias of that estimate. 2/

1/ For a description of methods of adjusting recorded births and deaths see Section E, Chapter VI

2/ When migration or natural increase is estimated as a residual and when alternate, comparable estimates are available, it must be remembered that the residual amounts will contain the total error of the estimating procedure. This is called the "errors of closure" which means the residual will include any estimating errors associated with overstatements or understatements of the components used to isolate the residual. Therefore, if the migration or natural increase alternate estimate does not correspond identically to the residual as described above, the estimator must remember that allowances are necessary for "errors of closure."

Natural Increase and Migration - Recorded Data

As an illustration of the component method just described, assume that a statistician is required to produce estimates of natural increase and migration between two counts broken down to their age and sex elements. If the data at hand make it simpler to treat net migration as a residue, he may proceed as in Worksheet 3 which follows. If, on the other hand, the available data are such that natural increase is best treated as a residue, he may proceed as in Worksheet 4. In both instance, the period between the two censuses is assumed to be five years, the first census being taken on January 1, 1950 and the second on January 1, 1955.

Instructions for completing follow each of the worksheets.

WORKSHEET 3

COMPUTING THE COMPONENTS OF POPULATION CHANGE BETWEEN TWO CENSUSES WITH MIGRATION AS RESIDUE (MALE)

Age Groups	Male Population 1950 Census	Deaths to Male 1950 Population 1950-1955	1955 Expected Male Population 1/	Male Population 1955 Census	Male Net Migration 1950-1955
(1)	(2)	(3)	(4)	(5)	(6)
Less than 5	22,338	290	25,416 2/	25,011	- 405
5 through 9	21,556	65	22,048	21,661	- 387
10 " 14	20,034	84	21,491	21,323	- 168
15 " 19	19,258	154	19,950	19,291	- 659
20 " 24	17,899	160	19,104	18,063	-1,041
25 " 29	15,883	175	17,739	16,317	-1,422
30 " 34	14,246	199	15,708	14,766	- 942
35 " 39	13,448	240	14,047	13,626	- 421
40 " 44	12,861	360	13,208	13,114	- 94
45 " 49	11,843	509	12,501	12,420	- 81
50 " 54	10,064	648	11,334	11,317	- 17
55 " 59	8,150	776	9,416	9,398	- 18
60 " 64	6,868	920	7,374	7,494	+ 120
65 " 69	5,596	1,086	5,948	6,240	+ 292
70 " 74	4,670	1,410	4,510	4,628	+ 118
75 and over	5,011	2,861	5,410	5,437	+ 27
TOTAL (Σ)	209,725	9,937	225,204	220,106	-5,098

1/ Excludes the effect of net migration. 2/ Male births, 1950-1955 (26,256) - deaths to male children born 1950-1955 (840) = expected male population "less than 5", 1955 (25,416).

Instructions for computing the components of population change between two censuses with migration as residue. An identical worksheet should be developed for females.

Column (1): List age groups in brackets equal, preferably, to the inter-censal period. 1/ In the present case, the age groups are listed in five year brackets.

Column (2): Opposite each age bracket, list the corresponding count from the census of 1950. 2/

Column (3): This column represents the number of deaths for each age group listed in (2) over the inter-censal period, 1950-1955, that is, after January 1, 1950 and before December 31, 1955. Each death can be attributed to the proper age group if the exact age of the deceased is known and also the date of his death. For instance, a person dying at the age of 46 years and three months in April of 1951 should be attributed to the age group "40 through 44" while a person of the same age dying in February, 1951 should be accredited to the age group "45 through 49." In practice, however, a close approximation of the number of deaths by age group may be arrived at more simply. 3/ For example, the deaths attributable to the age group "35 through 39" are the sum of:

1/ These age groups may be listed in brackets equal to a factor of the length of the inter-censal period. For example, in the case of a ten year period, the age groups may be listed in two or five year brackets. Any other bracket lengths would involve unnecessary computational difficulties. For breaking ten year age groups into five year brackets see Section A, Chapter VI.

2/ Adjustments of the census for misstatement of age and miscounts is necessary if there is reason to believe that a significant degree of inaccuracy in reporting the numbers in the different age groups is involved. For methods of adjusting census counts, see Section D, Chapter VI.

3/ Methods of testing and adjusting the total of recorded deaths are explained in Section E, Chapter VI.

(1) All the deaths at the ages 35 through 39 in 1950
(2) All the deaths at the ages 36 through 40 in 1951
(3) All the deaths at the ages 37 through 41 in 1952
(4) All the deaths at the ages 38 through 42 in 1953
(5) All the deaths at the ages 39 through 43 in 1954 1/

A similar procedure would apply for the other five-year age groups. For the age group "75 and over", the formula becomes the sum of:

(1) All the deaths at ages 75 and over in 1950
(2) All the deaths at ages 76 and over in 1951
(3) All the deaths at ages 77 and over in 1952
(4) All the deaths at ages 78 and over in 1953
(5) All the deaths at ages 79 and over in 1954 2/

Column (4): Subtract the figures in column (3) from the corresponding figures in column (2). Place the result of each subtraction one row below the original figures. For example, for the row "less than 5" place the result of (2)-(3) in (4) to correspond with the row "5 through 9" in (1). The reason for this is that by the time all the deaths in (3) occur among the original population, each group would have moved to the higher age bracket.

1/ A closer approximation may be obtained by taking the sum of:

(1) $1/2\ D_{35} + D_{36-39} + 1/2\ D_{40}$ in 1950.
(2) $1/2\ D_{36} + D_{37-40} + 1/2\ D_{41}$ in 1951.
(3) $1/2\ D_{37} + D_{38-41} + 1/2\ D_{42}$ in 1952.
(4) $1/2\ D_{38} + D_{39-42} + 1/2\ D_{43}$ in 1953.
(5) $1/2\ D_{39} + D_{40-43} + 1/2\ D_{44}$ in 1954.

Where Dx is the number of deaths to persons aged x years.

This method presumes that deaths are distributed uniformly over each year. Notice that the deaths attributed to the cohort in 1950 by this method will be different than the first method by the amount that $1/2\ D_{40}$ is different from $1/2\ D_{35}$. The use of this method, however, involves additional computational difficulties while contributing little to the accuracy of the results.

2/ The formula for the age group "75 and over" corresponding to footnote 1/ is the sum of $1/2\ D_{75} + D_{76}$ and over in 1950 etc., and the result is smaller than that of the formula in the text by $1/2\ D_{75}$ in 1950, $1/2\ D_{76}$ in 1951, $1/2\ D_{77}$ in 1952, $1/2\ D_{78}$ in 1953 and $1/2\ D_{79}$ in 1954.

Obviously, the first row in column (4) would be left empty following this procedure. It should be filled by the figure for actual live births between 1950 and 1955 minus the deaths occurring to that group during the period, as shown in footnote 2/ of Worksheet 3.

Column (5): List opposite each age group in column (1) the actual population in 1955. 1/

Column (6): For each row, subtract column (4) from column (5) to obtain net migration for the specific age group. If the expected population in (4) is larger than the corresponding population in (5), the sign will be negative indicating out-migration for that age group. If the reverse is true, the sign will be positive indicating in-migration.

The summations (Σ) of columns (2) through (6) are significant:

Σ (2) = Actual population in 1950.
Σ (3) = Total deaths, 1950-1955.
Σ (4) = The approximate size of total population in 1955 if no net migration had taken place.
Σ (5) = Actual population in 1955.
Σ (6) = Net migration.

By subtracting Σ (2) from Σ (5), we obtain (ΔP), i.e., population change, 1950-1955. Moreover, Σ (4) - Σ (2) gives the value of (N_i), i.e., natural increase. Finally, Σ (6), as we have already mentioned, gives the value of (M), or migration. As a check on our calculations,

$$\Delta P = N_i + M$$

therefore:

$$\Sigma(5) - \Sigma(2) = \Sigma(4) - \Sigma(2) + \Sigma(6)$$

Hence:

$$220,106 - 209,725 = 225,204 - 209,725 + (-5,098) \therefore 10,381 = 10,381$$

1/ The census count should also be adjusted if necessary (see Section D, Chapter VI).

WORKSHEET 4

COMPUTING THE COMPONENTS OF POPULATION CHANGE BETWEEN TWO CENSUSES WITH NATURAL INCREASE AS RESIDUE

MALE

Age Groups	Male Population 1950	Male Population 1955	(ΔP) Population Change 1950-1955	Male In-Migration 1950-1955	Male Out-Migration 1950-1955	(M) Net-Migration 1950-1955	(N_1) Natural Increase 1950-1955
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Less than 5	22,338	25,011	25,011	143	548	-405	+25,416
5 through 9	21,556	21,661	-677	122	509	-387	-290
10 " 14	20,034	21,323	-233	111	279	-168	-65
15 " 19	19,258	19,291	-743	163	822	-659	-84
20 " 24	17,899	18,063	-1,195	437	1,478	-1,041	-154
25 " 29	15,883	16,317	-1,582	616	2,038	-1,422	-160
30 " 34	14,246	14,766	-1,117	589	1,531	-942	-175
35 " 39	13,448	13,626	-620	196	617	-421	-199
40 " 44	12,861	13,114	-334	103	197	-94	-240
45 " 49	11,843	12,420	-441	76	157	-81	-360
50 " 54	10,064	11,317	-526	27	44	-17	-509
55 " 59	8,150	9,398	-666	26	44	-18	-648
60 " 64	6,868	7,494	-656	136	16	+120	-776
65 " 69	5,596	6,240	-628	321	29	+292	-920
70 " 74	4,670	4,628	-968	131	13	+118	-1,086
75 and over	5,011	5,437	-4,244	35	8	+27	-4,271
TOTAL	209,725	220,106	10,381	3,232	8,330	-5,098	15,479

Instructions for completing Worksheet 4 to compute the components of population change between two censuses with Natural Increase as residue. A similar worksheet should be completed for females.

Worksheet 4 is very similar to Worksheet 3. Note, for example, that in both cases the last column is the "residue." Hence, the last column in Worksheet 3 refers to migration while the last column in Worksheet 4 refers to natural increase.

Column (1): As in Worksheet 3, list age groups in brackets equal to the inter-censal period.

Column (2): Opposite each age bracket list the corresponding count from the census of 1950. 1/

Column (3): As in (2), list the 1955 population by age.

Column (4): In order to obtain the change in population for each age group, subtract each 1955 age group from the previous age group in 1950. The result indicates the number of people gained or lost in each age group while reaching the next age group five years later. The first row in (4) corresponding to the age group "less than 5"--is left empty following this procedure. It should, however, reflect the addition to total population caused by birth between 1950 and 1955, discounted by the death to this age group and net migration. This is nothing but the age group "less than 5" as reported in 1955. Hence, the first row figure of column (4) is the same as that of column (3).

Furthermore, in listing the results of the subtractions, column (4) will necessarily have one row for the "80 and over" age group. This should be added to the figure corresponding to the age group "75 and over." 2/

1/ Adjusted if necessary (see Section D, Chapter VI).

2/ The reader should work out the examples given in this section very carefully in order to better understand the different steps in the worksheets.

Column (5): From the migration records--adjusted if necessary--list the number of in-migrants in the appropriate age groups. As with deaths in Worksheet 3, it is necessary here to know the age, sex, and date of entry of each in-migrant. A 33 year old male in-migrant, for example, would not necessarily be listed in the row corresponding to the age group "30 through 34" in Worksheet 4. If he in-migrated in, say, 1951, he will be 37 years old in 1955, and should be accredited to the age group "35 through 39." The proper age group is, therefore, the age group where the in-migrant belongs at the date of the second census; namely, 1955.

Column (6): As in column (5), list in this column the number of out-migrants in the appropriate age--sex groups.

Column (7): For each row, subtract the figure in column (6) from that in column (5). Where the former figure is larger than the later, the result will be negative denoting net out-migration. In the opposite case, the result will be positive denoting net in-migration.

The totals of columns (2) through (8) signify the following:

Σ (2) = actual population in 1950.
Σ (3) = actual population in 1955.
Σ (4) = change in population between the two dates, i.e., $\Sigma (4) = \Sigma (3) - \Sigma (2)$.
Σ (5) = number of in-migrants.
Σ (6) = number of out-migrants.
Σ (7) = net migration, i.e., $\Sigma (7) = \Sigma (6) - \Sigma (5)$.
Σ (8) = natural increase, i.e., $\Sigma (8) = \Sigma (4) - \Sigma (7)$.

Again, the computations may be checked by applying the data to the equation:

$$\Delta P = N_i + M$$

$$220,106 - 209,725 = 15,479 + (-5,098)$$

$$10,381 = 10,381$$

Natural Increase and Migration - Survival Ratios

The component method for estimating natural increase and migration between two censuses described in the previous section makes use of recorded data of either natural increase (births and deaths) or migration (in-migrants and out- migrants). This recorded data, however, may either be inadequate or completely unavailable in the breakdowns required. Furthermore, even when these data are available and reliable, their compilation may prove cumbersome to the estimator. 1/ Therefore, an alternative component method utilizing survival ratios for the natural increase component but otherwise similar to the method described above will be outlined below.

Following this method, migration is treated as residue and natural increase estimated partially or totally through theoretical devices. More specifically, the population enumerated at the first census, including live births between the two censuses, is adjusted by "survival ratios" (i. e., the probability of survival to the next census date) to obtain the expected total population at the second census if migration had not taken place. The difference between this result and the actual size of the population at the second census is then taken as an estimate of migration over the period. If the expected population is larger than the actual population, net out-migration is indicated while if the actual population is larger than expected population, net in-migration would have taken place.

When migration is to be estimated by age and sex, the survival ratios would have to be obtained by age and sex. 2/ These survival ratios are usually obtained from a life table reflecting the mortality conditions in the area at the appropriate time. 3/ For the present chapter, it is

---

1/ See description of column (3) of Worksheet 3 and columns (5) and (6) of Worksheet 4 of this chapter.

2/ In fact, whatever breakdown is required for migration, a similar breakdown is required for the survival ratios.

3/ See Section C, Chapter VI, for construction of life tables and for other survival ratios.

assumed that the appropriate survival ratios are given and a step by step procedure of computing the growth components of population change between two censuses is explained in the instructions for completing Worksheet 5.

WORKSHEET 5

COMPUTING THE COMPONENTS OF POPULATION CHANGE BETWEEN TWO CENSUSES

(MALES)

Age Group	Male Population 1950 Census	5-Year Survival Ratios	Expected Male Population 1955	Actual Male Population 1955 Census	Net Male Migration 1950-1955
(1)	(2)	(3)	(4)	(5)	(6)
Born 1950-55	26,256	0.9701			
Less than 5	22,338	0.9891	25,471	25,011	-460
5 through 9	21,556	0.9981	22,095	21,661	-434
10 " 14	20,034	0.9972	21,515	21,323	-192
15 " 19	19,258	0.9936	19,978	19,291	-687
20 " 24	17,899	0.9914	19,135	18,063	-1,072
25 " 29	15,883	0.9900	17,745	16,317	-1,428
30 " 34	14,246	0.9870	15,724	14,766	-958
35 " 39	13,448	0.9841	14,061	13,626	-435
40 " 44	12,861	0.9728	13,234	13,114	-120
45 " 49	11,843	0.9600	12,511	12,420	-91
50 " 54	10,064	0.9367	11,369	11,317	-52
55 " 59	8,150	0.8992	9,427	9,398	-29
60 " 64	6,868	0.8421	7,328	7,494	+166
65 " 69	5,596	0.7922	5,784	6,240	+456
70 " 74)	9,681	0.5545	4,433	4,628	+195
75 and over)			5,368	5,437	+69
Total	209,725 1/		225,178	220,106	-5,072

1/ Excludes births 1950-55.

Worksheet 5

Reserve the first row for births during the period 1950-1955.

Column (1): List age groups in brackets equal to the inter-censal period. 1/ In the present case, the age groups are listed in five year brackets.

Column (2): Opposite each age bracket, list the corresponding count from the census of 1950 (beginning of period). In the first row, list births between 1950 and 1955. 2/

Column (3): List the probability of five year survival for each age group (from appropriate life table). 3/

Column (4): For each row, multiply the probability of survival by the initial age group--i.e., (3) x (2)--placing the result one row below in (4) to indicate the expected survivors five years later, i.e., until reaching the next age bracket.

Column (5): List opposite each age group of column (1) the actual population in 1955.

Column (6): Subtract column (4) from column (5) for each row to obtain net migration by age group. A negative sign indicates net out-migration and a positive sign net in-migration.

1/ See footnote 1/, page 74.

2/ The figure for the first row indicates actual live births between the two censuses. This may be secured either from vital statistics records or through estimates of the birth rate. Estimating births is discussed in Section B, Chapter VI.

3/ See Section C, Chapter VI for Computing Survival Ratios.

The relevant column totals (Σ) are the following:

Σ (2) = Total male population in 1950.
Σ (4) = Expected population in 1955 if no migration took place.
Σ (5) = Actual population in 1955.
Σ (6) = Net male migration 1950-1955.

Hence:

$\Sigma (5) - \Sigma (2) = \Delta P$ = Population change
$\Sigma (4) - \Sigma (2) = N_i$ = Natural increase
$\Sigma (5) - \Sigma (4) = M$ = Net migration.

In estimating the components of population change between two census counts, it is essential that the basic data used in carrying out the calculations be of a relatively high degree of accuracy. The reason for this is simple. A small percentage error in the size of an age group in one census, without a compensatory error in the size of that cohort at the time of the next census, may easily turn in-migration to out-migration for that age group. As an illustration, take the 1950 age group "40 through 44" in Worksheet 3 where the number of males reported in that age group is 12,861. Five years later, this number had diminished by 441 persons, thus the age group "45 through 49" reported in 1955 was 12,420 persons. It is known from the vital statistics data, which are assumed to be reliable, that only 360 persons of the initial 12,861 died during the period. Hence, it is concluded that 81 persons must have out-migrated. Assume, however, that the 1955 count of the age group "45 through 49" was an under-count, and that the true size of this age group in 1955 was actually one percent higher than reported, namely, 12,544. The actual loss in population is, therefore, 317 persons. This is, in fact, larger than the loss from deaths to that age group between 1950 and 1955 by 43 persons. While we had concluded, on the basis of the slight under-count, that 81 persons among that group out-migrated during the period, there were instead 43 persons belonging to that age bracket who had in-migrated.

As has often been repeated in population studies, it is necessary to emphasize caution in interpretation. Where heavy in-migration or heavy out-migration is indicated, the exact amount may remain in doubt but there will seldom be reason to question the direction of migration or the fact it was considerable. On the other hand, estimates of small gains or losses through migration should be regarded more as establishing the fact that change through migration was not important rather than as an indicator of either the magnitude or the direction of migration. 1/

It should be kept in mind, therefore, that, even after all basic data are adjusted as far as possible, caution in interpretation still remains in order. This point cannot be overemphasized. With it in mind, a slightly different situation will be explored in the next chapter. There, the components of population change will be projected beyond the date of the last Census.

1/ E.S. Lee, et. al., Population Redistribution and Economic Growth, United States, 1870-1950, Vol. I, The American Philosophical Society, Philadelphia, 1957, p. 34.

PART II
CHAPTER III

USE OF THE COMPONENT METHOD FOR POPULATION PROJECTION

In the previous chapter, it was stated that the relation between population change (ΔP) and its growth components -- natural increase (N_i) and migration (M)--is given by the following equation:

$$\Delta P = N_i + M$$

It was also stated that, since ΔP is known when dealing with a period between two counts, either N_i or M had to be secured or estimated. However, in the case of projecting population into the future, ΔP is not available. When projecting, therefore, all three variables in the equation are unknowns. The values of any two of them are necessary and sufficient to solve for the third variable which then may be considered as "residue." For example, if the values of ΔP and N_i are estimated, their difference will constitute an estimate of M. If N_i and M are first estimated, their sum will constitute an estimate of ΔP.

The first question that may face the statistician at this point is which two variables are to be estimated first. In other words, in making a component projection of total population, a decision has to be made as to what two variables are to be considered as determinants, and, hence, estimated first and what variable is to be considered as determinate, and computed as residue. Obviously, the statistician's choice among the different alternatives may be heavily weighted by the availability and reliability of the relevant data. But there is more to his choice than this. It is necessary to find out which variables actually are, in the instance at hand, primarily determined by factors independent of the other variables in the population equation, and which remaining variable is mainly determined by the interaction of the other two.

It may seem, at first sight, that population change is almost by definition "determined" by natural increase and migration so that it may ideally be treated as residue. But this is not true in all cases. There are obvious instances when over-all population growth is, to a major extent, determined by the rate of economic growth--more exactly, perhaps, by the rate of expansion in job opportunities--while the rate of natural increase is determined mainly by institutional factors affecting the levels of births and deaths. In such instances, migration, not population change, may be properly considered as determinate (or residue) in the population equation. [1]

The Mechanics of Component Projections

From this brief introduction, two procedures for making component projections of total population suggest themselves. [2] In the first procedure, natural increase and net migration are first projected by age, and their total taken as an estimate of population change by age group. [3] In the second procedure, natural increase and population change are first projected, and their difference taken as an estimate of net migration.

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[1] The author in, *Population, Labor Force, and Income Trends in the Knoxville Area and Tennessee*, Tennessee Department of Employment Security, Nashville, 1961, used a variation of the component method that treats migration as residue in projecting the population of the State of Tennessee from 1960 to 1970. While this method was much less elaborate than the U.S. Census Bureau method used later to make the same projection, the results of both methods were strikingly similar.

[2] A third procedure may be one that considers natural increase as residue. This procedure, however, may hardly be recommended here because it is recognized that, at least in the short-run, natural increase seems to be most independently determined of the three relevant variables.

[3] If age and sex breakdowns are required, one worksheet for each sex will be necessary. The basic technique, however, will not change and, for the sake of clarity, our illustrations will be made without reference to sex.

a. The First Procedure:

The basic technique for the first procedure may be summarized in the following steps:

1. Discount each age group, including expected births during the projection period, by an appropriate survival ratio 1/, to obtain the expected size of each age group, exclusive of migration at the end of the period.
2. Estimate net migration by age group.
3. Add the results of 1. and 2. for each age group to obtain an estimate of the net size of each age group at the end of the projection period.

b. The Second Procedure:

The basic technique for the second procedure may be summarized in the following steps:

1. Discount each age group, including expected live births during the projection period, by an appropriate survival ratio, to obtain the expected size of each age group, exclusive of migration, at the end of the period.

1/ If no appropriate life table is found, see Section C, Chapter VI for methods of constructing such tables.

2. On a separate sheet: (a) project total population by an appropriate extrapolation method [1] and subtract the initial population total from the projected total to obtain an estimate of population change (ΔP). (b) The difference between initial population and the discounted population of step 1 constitutes an estimate of natural increase (N_i). Hence, again on a separate sheet, find $\Delta P - N_i$, which may be taken as an estimate of net migration. (c) From past migration records, or model age distribution of migration, estimate the age distribution of migration for the projection period and apply these ratios to the estimated net migration of step 2. Insert these estimates in the proper place, in the master worksheet.

3. Add, for each age group, N_i and M to obtain an estimated population change (ΔP) by age group.

A little reflection on the part of the reader should make it clear that the master worksheets of both procedures look exactly alike (see Worksheet 6 below). The difference between them is essentially in the method of obtaining the estimates of M, net migration (Col. 5) and P, estimated population in the future year, (Col. 6). For illustration purposes, therefore, only one model worksheet will be used to explain the techniques of both procedures. Instructions for completing the worksheet are followed by detailed instructions for developing necessary worksheet data through the application of both the first and the second procedures mentioned earlier.

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[1] e.g. see extrapolation methods of Part I Chapter III.

WORKSHEET 6

PROJECTING THE
GROWTH COMPONENTS OF
POPULATION CHANGE

(MALE)

Age Groups	Population 1960 Census	5-Year Male Survival Ratios	1965 Population Excl. Migration	Net Migration 1960-1965	Estimated Population 1965
(1)	(2)	(3)	(4) = (2) x (3)	(5)	(6) = (4+5)
Less than 5	212,356	0.9869	217,755[1/]	-4,049	213,706
5 through 9	205,480	0.9969	209,574	-3,795	205,779
10 " 14	200,651	0.9953	204,843	-1,559	203,284
15 " 19	190,395	0.9919	199,708	-7,496	192,212
20 " 24	176,861	0.9904	188,853	-10,669	178,184
25 " 29	157,807	0.9891	175,163	-13,917	161,246
30 " 34	141,505	0.9860	156,087	-9,138	146,949
35 " 39	130,440	0.9804	139,524	-4,134	135,390
40 " 44	124,289	0.9716	127,883	-937	126,946
45 " 49	113,377	0.9574	120,759	-786	119,973
50 " 54	92,355	0.9353	108,547	-140	108,407
55 " 59	78,472	0.9044	86,380	-172	86,208
60 " 64	62,428	0.8654	70,970	+1,380	72,350
65 " 69	51,937	0.8061	54,025	+3,108	57,133
70 " 74	39,580	0.6987	41,866	+1,232	43,098
75 and over	46,778	0.4865	50,412	+262	50,674
Total	2,024,711		2,152,349	-50,810	2,101,539

[1/] Estimated Births 1960-1965 = 225, 000: Survival ratio for this group = 0. 9678

Instructions for Completing Worksheet 6

Column (1): List age groups in brackets equal to the projection period. 1/

Column (2): Opposite each age bracket, list the corresponding count from the census of 1960. 2/

Column (3): Opposite each age group, list the estimated proportion of survivors to the end of the period. These percentages, called survival ratios, may be obtained from an appropriate life table. 3/

Column (4): For each row, multiply column (2) by column (3) to obtain the estimated number of survivors that reached the next age group. List the result in column (4) one row below the multiplication row in columns (2) and (3). It follows that the first row will be left empty while one extra row will be created at the bottom. The empty row must be filled with the

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1/ If age groups are listed in ten year brackets and the projection period extends over only five years, halving may become necessary. One formula to use in such an instance is given in Section A, Chapter VI. If the age groups are listed in five year brackets while the projection period is slightly longer than five years, say six years from the date of the last census, it may be advisable to move the age distribution one year forward. Methods accomplishing this end are to be found in the Methods of Population Projections by Age and Sex, (ST/SOA/Series A, Population Studies No. 25 by the United Nations.)

2/ Adjusted if necessary. Methods of adjustment are outlined in Section E Chapter VI.

3/ If no suitable life table is available, consult the short-cut method of building life tables in Chapter VI, Section C estimated number of births [1] during the projection period discounted by their survival ratio, also secured from the appropriate life table. The extra row at the bottom which refers to the number of survivors 80 years old and over must be added to the previous row which now refers to the survivors reaching the ages between 70 and 75 to make the result an estimate of the survivors beyond the age of 75 years.

Column (5): List here opposite each age group the estimated net migration for that age group during the period. Methods for making these estimates (projection migration) follow the instruction for completing this worksheet (worksheet 6). Net out-migration is denoted by a minus sign and net in-migration by a plus sign. For the age group "less than 5" list the estimated migration of those born between 1960 and 1965, for the age group "5 through 9" list the estimated migration of the 1960 "less than 5" age group and so on.

Column (6): For each row, add algebraically--i.e., subtracting, when the net migration sign is negative and adding when it is positive--columns (4) and (5) to obtain an estimate of 1965 population by age group.

The totals (Σ) of columns (2), (4), (5) and (6) mean the following:

Σ (2) = Actual total population in 1960.
Σ (4) = Approximate total population if migration had not taken place.
Σ (5) = Estimated net migration.
Σ (6) = Estimated population in 1965.

Now, $\Sigma (6) - \Sigma (2) = \Delta P$ and as a check on our calculations, we must find that $\Delta P = N_i + M$ or:

$$\Sigma (6) - \Sigma (2) = [\Sigma (4) - \Sigma (2)] + \Sigma (5)$$

$$76{,}828 = 76{,}828$$

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[1] Future births may be estimated by projecting past birth data. See Section B Chapter VI.

Projecting Migration

The only column in Worksheet 6 that needs to be elaborated on here is column (5), net migration. This section contains methods for computing migration by age, utilizing each of the two previously mentioned procedures. However, before doing this, it might be advantageous to clearly define the difference between two sets of migration rates; (1) the age-specific rates, and (2) the rates giving the relative age distribution of migration. [1/]

The age-specific rate relates the number of net migrants in each age group during a given period to the total number of persons in that age group at the beginning of the period. For example, in Worksheet 6 the number of people in the age group "15 through 19" is 190, 395 at the beginning of the period (1960) and the net number of migrants from that age group, during the period, is estimated at - 10, 669 persons (the minus sign denoting net out-migration.) The age-specific migration rate for that cohort during the period is $\frac{-10,669}{190,395} = -0.0560$ or an out-migration of 5.6 percent. On the other hand, the percentages giving the relative age distribution of migration relate the net number of migrants in each age group over a period of time to total net migration. For example, if the net number of migrants of the 1960 "15 through 19" group is again -10, 669 and total net migration, i. e., the total of column (5) in Worksheet 6 above, is -50, 810, then the percentage showing the relative migration of this age group is $\frac{-10,669}{-50,810} = 0.2100$ or 21 percent. In other words, 21 percent

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[1/] Note here again that if the sex breakdown is also required, age-sex specific rates should be substituted for age-specific rates. Age-sex specific migration rates relate to age-sex groups (e. g. Males, "15 through 19".) Also relative age-sex distribution will have to be considered instead of relative age distribution. For the sake of simplicity we have neglected the sex component in this chapter.

of the total net out-migrants are "15 through 19" years of age at the beginning of the period or "20 through 24" at the end of the period. [1]

Note again that the difference between the two procedures is simply that, in the first, migration and natural increase are projected separately for each age group and their addition taken as an estimate of population change for that age group while, in the second, total population change and natural increase are first projected and total migration considered to be equal to their difference. In the first procedure, therefore, age-specific migration rates are projected while in the second, projections are made for total migration. It follows, hence, that a further step is required in the latter procedure; namely, the breaking down of total migration to its age components.

It should be obvious now that the first procedure makes use of age-specific migration rates (or magnitudes) while the second procedure makes use of the rates given by the relative age distribution of migration.

First Procedure for Projecting Migration

This procedure involves projection into the future of past age-specific migration trends. If absolute age-specific magnitudes are projected, computations will be similar to those in Worksheet 7 which follows. If rates of migration of the different age groups are projected, computations will be similar to those in Worksheet 8 or 9 depending on the availability and reliability of the basic data.

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[1] It is customary to list the age-specific rate opposite the age group of the cohort at the beginning of the period while the percentage given by the relative age distribution is listed opposite the age of the cohort at the end of the period. As will be noticed, the reason for this is to eliminate some computational difficulties in the worksheet.

Note at the outset, however, that while the computations in Worksheet 7 are simpler than in either of the two other worksheets, the results are likely to be less reliable. When absolute magnitudes of age-specific migration are projected, the accuracy of the result will depend on the method of extrapolation and on the net number of migrants. It is, hence, independent of the change in the size of the age group itself. Situations may arise, therefore, where migration of a given age group is presumed to increase while the age group itself is decreasing, thus increasing the rate of migration of that age group to unjustifiable levels. The results of this method, however, become more reliable when the age distribution of the population is fairly smooth.

WORKSHEET 7

PROJECTING ABSOLUTE LEVELS OF AGE-SPECIFIC MIGRATION

			EXTRAPOLATION		
AGE GROUP	NET MIGRATION 1950-55	NET MIGRATION 1955-60	LATEST Magnitude ('55-'60) I	AVERAGE Magnitude ('50-55 and '55 to '60) II	ARITHMETIC Extrapolation III
(1)	(2)	(3)	(4)	(5)	(6)
Born during period	-3,537	-4,005			
Less than 5	-2,966	-3,756	-4,005	-3,771	-4,473
5 through 9	-1,016	-1,561	-3,756	-3,361	-4,546
10 " 14	-6,201	-7,534	-1,561	-1,289	-2,106
15 " 19	-8,992	-10,293	-7,534	-6,868	-8,867
20 " 24	-12,431	-13,879	-10,293	-9,643	-11,594
25 " 29	-8,006	-9,037	-13,879	-13,155	-15,327
30 " 34	-3,412	-4,040	-9,037	-8,522	-10,068
35 " 39	-766	-891	-4,040	-3,726	-4,668
40 " 44	-687	-752	-891	-829	-1,016
45 " 49	-102	-140	-752	-720	-817
50 " 54	-100	-157	-140	-121	-178
55 " 59	+544	+1,186	-157	-129	-214
60 " 64	+1,381	+2,609	+1,186	+865	+1,828
65 " 69	+987	+1,036	+2,609	+1,995	+3,837
70 " 74	+88	+112	+1,036	+1,012	+1,085
75 and over	+100	+135	+247	+218	+306
TOTAL	-45,116	-50,967	-50,967	-48,044	-56,818

Instructions for Completing Worksheet 7

Projecting absolute levels of age-specific migration:

Column (1): List age group in brackets equal to the projection period. Denote the first row by "born during period."

Columns (2) and (3): If fairly accurate records of migration are kept, compute migration for each age group during periods equal to the age brackets, (in this instance 5 years) as explained in the instructions for completing columns (5), (6), and (7) of Worksheet 4 (page 77). If migration records are non-existent, past migration by five year age groups may be estimated as in Worksheet 3 (page 73). If data are so deficient that neither alternative is possible, it may become advisable to use the method of the second procedure below. 1/

Columns (4), (5) and (6): These represent the results of different methods of extrapolation. 2/ Column (4) assumes the same age-specific migration magnitudes of the last five years to apply to the next five years or projection period. Column (5) assumes that the average of the last two 5 year periods will apply to the next 5 years. Column (6) is obtained by extrapolating arithmetically the last two five year periods. Figures in all three columns are dropped one row to correspond to 1965, i.e., the end of the projection period.

1/ See introduction to this chapter for criteria for choice between these two alternatives.

2/ Other methods of extrapolation may be used. Other extrapolation techniques are explained in Part I Chapter III.

WORKSHEET 8

PROJECTING AGE - SPECIFIC MIGRATION RATES

AGE GROUP	NET MIGRATION 1950-55	NET MIGRATION 1955-60	POPULATION 1950	POPULATION 1955	POPULATION 1960	AGE-SPECIFIC MIGRATION RATES 1950-55 (% of "50)	AGE-SPECIFIC MIGRATION RATES 1955-60 (% of '55)	PROJECTED MIGRATION RATIOS LATEST RATIOS 1/ I	PROJECTED MIGRATION RATIOS AVERAGE RATIOS II	PROJECTED MIGRATION RATIOS ARITHMETIC EXTRAPOLATION RATIOS III	NET MIGRATION 1960 - 1965 I	NET MIGRATION 1960 - 1965 II	NET MIGRATION 1960 - 1965 III
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
						(2) ÷ (4)	(3)÷(5)				(9) X (6)	(10) X (6)	(11) X (6)
Born during period	-3,537	-4,005	213,661 2/	216,361 2/	217,755 2/	-.01655	-.01851	-.01851	-.01753	-.02047			
Less than 5	-2,966	-3,756	209,724	210,124	212,356	-.01414	-.01788	-.01788	-.01601	-.02162	-4,031	-3,817	-4,457
5 through 9	-1,016	-1,561	204,500	204,011	203,615	-.00497	-.00765	-.00765	-.00631	-.01033	-3,797	-3,400	-4,591
10 " 14	-6,201	-7,534	194,646	202,850	201,818	-.03186	-.03714	-.03714	-.03450	-.04242	-1,558	-1,285	-2,103
15 " 19	-8,992	-10,293	185,729	187,530	194,363	-.04841	-.05489	-.05489	-.05165	-.06137	-7,496	-6,963	-8,561
20 " 24	-12,431	-13,879	171,987	175,233	175,718	-.07228	-.07920	-.07920	-.07574	-.08612	-10,669	-10,039	-11,928
25 " 29	-8,006	-9,037	153,466	157,905	159,672	-.05217	-.05723	-.05723	-.05470	-.06229	-13,917	-13,309	-15,133
30 " 34	-3,412	-4,040	136,204	143,787	147,147	-.02505	-.02810	-.02810	-.02658	-.03115	-9,138	-8,734	-9,946
35 " 39	-766	-891	125,101	130,885	137,734	-.00612	-.00681	-.00681	-.00647	-.00750	-4,135	-3,911	-4,584
40 " 44	-687	-752	121,897	121,883	127,429	-.00564	-.00617	-.00617	-.00591	-.00670	-938	-891	-1,033
45 " 49	-102	-140	107,621	117,748	117,670	-.00095	-.00119	-.00119	-.00107	-.00143	-786	-753	-854
50 " 54	-100	-157	88,418	102,934	112,592	-.00115	-.00153	-.00153	-.00133	-.00193	-140	-126	-168
55 " 59	+544	+1,186	69,845	82,597	96,117	+.00779	+.01436	+.01436	+.01108	+.02093	-172	-150	-217
60 " 64	+1,381	+2,609	54,516	63,712	75,887	+.02533	+.04095	+.04095	+.03314	+.05657	+1,380	+1,065	+2,012
65 " 69	+987	+1,036	46,187	48,559	57,745	+.02137	+.02133	+.02133	+.02135	+.02129	+3,108	+2,515	+4,293
70 " 74	+88	+112	36,314	38,218	40,179	+.00242	+.00293	+.00293	+.00268	+.00344	+1,232	+1,233	+1,229
75 and over	+100	+135	42,721	46,345	49,497	+.00234	+.00291	+.00291	+.00263	+.00348	+262	+238	+310
TOTAL	-45,116	-50,967	1,948,876	2,034,321	2,109,539	-.02086	-.02505	-.02409	-.02292	-.02643	-50,795	-48,327	-55,731

1/ If latest age-specific migration ratios are deemed adequate, only columns (1), (3), (5), (6), (9), and (12) will be necessary.

2/ Live births; not included in total.

Instructions for Completing Worksheet 8

Projecting age-specific migration rates:

Columns (1), (2) and (3): Same as (1), (2) and (3) in Worksheet 7.

Columns (4), (5) and (6): List the 1950, 1955 and 1960 populations respectively. (If the 1955 population is not available use the procedure described in Worksheet 9). In the first row, list live births, for the five years succeeding the year for which the population is listed and for 1960 project future births (see Section B Chapter VI).

Column (7): Divide, for each age group, column (2) by column (4) to obtain the percentage of each age group migrating between 1950 and 1955 (i.e., the age-specific migration rates, 1950 - 1955.)

Column (8): Divide, for each age group, column (3) by column (5) to obtain the percentages of each age group migrating between 1955 and 1960, (i.e., the age-specific migration rates, 1955 - 1960.)

Columns (9), (10) and (11): Here the ratios in columns (7) and (8) are extrapolated to apply to 1960 - 65, following the same methods used in columns (4), (5) and (6) in Worksheet 7. Column (9) assumes the same age specific migration rate of the previous five years to apply to the projection period. Column (10) refers to the average of the age-specific rates of the two previous five year periods. Column (11) is obtained by extrapolating arithmetically the ratios in (7) and (8).

Column (12): This is the first series of estimates of net migration in this worksheet. It is obtained by multiplying column (6) by column (9) to obtain the estimated number of migrants in each age group for the projection period 1960 - 65.

Column (13): This is the second series of estimates. It is obtained by multiplying for each age group column (6) by column (10).

Column (14): This is the third series of estimates. It is obtained by multiplying column (6) by column (11).

Figures in each of the last three columns are dropped one row to correspond to the end of the projection period.

WORKSHEET 9

PROJECTING AGE - SPECIFIC MIGRATION RATES

AGE GROUP	NET MIGRATION 1950-55	NET MIGRATION 1955-60	POP. 1955 EXCLUDING MIGRATION	POP. 1960 EXCLUDING MIGRATION	POP. 1965 EXCLUDING MIGRATION	AGE-SPECIFIC MIGRATION RATES		PROJECTED MIGRATION RATIOS			NET MIGRATION 1960 - 1965		
						1950-55 (2)÷(4)	1955-60 (3)÷(5)	LATEST RATIOS	AVERAGE RATIOS	ARITHMETIC EXTRAPOLATION RATES	(6) X (9)	(6) X (10)	(6) X (11)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
LESS THAN 5	-3,537	-4,005	209,388	216,361	214,380	-.01689	-.01851	-.01851	-.01770	-.02031	-3,968	-3,795	-4,354
5 THROUGH 9	-2,966	-3,756	206,977	207,371	209,574	-.01433	-.01811	-.01811	-.01622	-.02189	-3,795	-3,399	-4,588
10 " 14	-1,016	-1,561	203,866	203,379	202,984	-.00498	-.00768	-.00768	-.00633	-.01038	-1,559	-1,285	-2,107
15 " 19	-6,201	-7,534	193,731	201,897	200,869	-.03201	-.03732	-.03732	-.03467	-.04263	-7,496	-6,964	-8,563
20 " 24	-8,992	-10,293	184,225	186,011	192,789	-.04881	-.05534	-.05534	-.05208	-.06187	-10,669	-10,040	-11,928
25 " 29	-12,431	-13,879	170,336	173,551	174,031	-.07298	-.07997	-.07997	-.07648	-.08696	-13,917	-13,310	-15,134
30 " 34	-8,006	-9,037	151,793	156,184	157,932	-.05274	-.05786	-.05786	-.05530	-.06298	-9,138	-8,734	-9,947
35 " 39	-3,412	-4,040	134,297	141,774	145,070	-.02541	-.02850	-.02850	-.02696	-.03159	-4,134	-3,911	-4,583
40 " 44	-766	-891	122,649	128,320	135,034	-.00625	-.00694	-.00694	-.00660	-.00763	-937	-891	-1,030
45 " 49	-687	-752	118,435	118,422	123,810	-.00580	-.00635	-.00635	-.00608	-.00690	-786	-753	-854
50 " 54	-102	-140	103,036	112,732	112,657	-.00099	-.00124	-.00124	-.00112	-.00149	-140	-126	-168
55 " 59	-100	-157	82,697	96,274	105,307	-.00121	-.00163	-.00163	-.00142	-.00205	-172	-150	-216
60 " 64	+544	+1,186	63,168	74,701	86,928	+.00861	+.01588	+.01588	+.01225	+.02315	+1,380	+1,065	+2,012
65 " 69	+1,381	+2,609	47,178	55,136	65,673	+.02927	+.04732	+.04732	+.03830	+.06537	+3,108	+2,515	+4,293
70 " 74	+987	+1,036	37,231	39,143	46,548	+.02651	+.02647	+.02647	+.02649	+.02643	+1,232	+1,233	+1,230
75 and over	+188	+247	46,157	49,250	52,153	+.00407	+.00502	+.00502	+.00455	+.00597	+262	+237	+311
TOTAL	-45,116	-50,967	2,075,164	2,160,506	2,225,739	-.02174	-.02359	-.02359	-.02267	-.02544	-50,729 1/	-48,308 1/	-55,626 1/

1/ Column total (not obtained by multiplying projected migration ratio by the total of column 6).

Instructions for Completing Worksheet 9

Projecting age-specific migration rates:

If no reliable counts or estimates are available for 1955 but are available for 1950 and 1960, the age specific ratios may then be related to 1955 (projected) population exclusive of migration and the 1960 population exclusive of migration.

Columns (1), (2) and (3): Same as in previous two worksheets except that the row for births is no more needed because we are now dealing with end of periods.

Column (4): Figures for "1955 population excluding migration" may be obtained by discounting the population of 1950 by appropriate survival ratios. [1]

Column (5): Figures for "1960 population excluding migration" may be obtained by subtracting the 1955-1960 migration from the 1960 population count of each age group. [2]

Column (6): Figures for "1965 population excluding migration" may be obtained by discounting the 1960 counts, including estimated live births between 1960 and 1965 [3], by appropriate survival ratios. [4]

Column (7) through (14): Same as in Worksheet 8.

---

[1] For securing these rates, see Section C, Chapter VI.

[2] If the 1955 counts were available and discounted by the appropriate survival ratios, the results would likely be slightly different than the results in column (5). The reason is not only that the survival ratios are only estimates of actual survival but also that they refer to the population living in the area at the beginning of the period, hence, take into account the mortality of the out-migrant and leave out the mortality of the in-migrants. For a detailed discussion of this aspect of the use of survival ratios see, E.S. Lee, et. al., Population Redistribution and Economic Growth United States, 1870-1950, Vol. 1, The American Philosophical Society, Philadelphia, 1957.

[3] For methods of projecting births, see Section B, Chapter VI.

[4] For securing these rates, see Section C, Chapter VI.

Second Procedure for Projecting Migration

As was already explained, total net migration over the projection period may be obtained as the difference between projected population change (ΔP) and projected natural increase (N_i). In order to apportion this migration to each age group in the population (i.e., in order to obtain the figures for column (5) in Worksheet 6 following the second population projection procedure) it is necessary to have a set of rates reflecting the probable age distribution of migration over the projection period. This set of percentages may be obtained in two ways:

If past relative age distributions of migration (records or estimates [1]) are available for levels (i.e., yearly rates) of migration comparable to the level of the estimated future migration [2], one set of these percentages, or a set obtained by averaging all percentages for each age group, may be applied to estimated total migration for the projection period to obtain the age specific migration magnitudes of column (5) of Worksheet 6. As an illustration assume that the size of total population of the hypothetical area of Worksheet 6 in 1955 is set at 1,943,734 while in 1960 it is set at 2,024,711 as shown on that worksheet. By straight geometric extrapolation the estimated 1965 population will be set at 2,109,748 persons. Projected population change between 1960 and 1965 will hence be equal to 2,109,748 - 2,024,711 = 85,037 (ΔP). Now, from Worksheet 6 we find that natural increase (N_i) between 1960-1965 may be estimated at 2,152,349 - 2,024,711 = 127,638 i.e. $\Sigma(4) - \Sigma(2)$. Total projected migration for 1960-1965 which is given by the equation $M = \Delta P - N_i$ is hence equal to 85,037 - 127,638 = -42,601 persons.

---

[1] A method estimating past migration by age and sex is presented in Part II Chapter II.

[2] The level of migration L_m over a given period t may be obtained by direct substitution in:

$$L_m = \frac{M}{P_o t}$$

where M is net migration total and P_o population total at the beginning of the period.

Assume now that the 1955-1960 relative age distribution of migration is available either from migration records or from estimates made following the methods described in Part II, Chapter II, and are found to be as in column (2) of Table IV. If each percentage is in turn applied to projected total migration (-42, 601) 1/, one will obtain the estimated age breakdowns of total migration as in column (3) of Table IV. This column may then be substituted for column (5) in Worksheet 6.

1/ The implied assumption here is that the relative age distribution of migration between 1960 and 1965 will be the same as it was between 1955 and 1960.

TABLE IV
APPORTIONMENT OF PROJECTED TOTAL MIGRATION TO AGE GROUPS WITH THE USE OF AN ASSUMED RELATIVE AGE DISTRIBUTION

Age Group (1)	Relative Age Distribution of Migration 1955-1960 (2)	Migration 1960-1965 (3)
Less than 5	- 5.5	- 2,343
5 through 9	- 7.9	- 3,365
10 " 14	- 5.1	- 2,173
15 " 19	- 14.8	- 6,305
20 " 24	- 21.0	- 8,946
25 " 29	- 27.4	-11,672
30 " 34	- 18.0	- 7,668
35 " 39	- 8.1	- 3,451
40 " 44	- 1.8	- 767
45 " 49	- 1.5	- 639
50 " 54	- .3	- 128
55 " 59	- .3	- 128
60 " 64	+ 2.7	+ 1,150
65 " 69	+ 6.1	+ 2,599
70 " 74	+ 2.4	+ 1,022
75 and over	+ .5	+ 213
Total	-100.0	-42,601

If no adequate past relative age distribution of migration is available, total estimated migration may be broken down to its age components with the proper percentages of Table V. 1/ If, for example, it is found that the level of migration is 1.25 percent a year, its relative age distribution may be assumed to correspond to the percentages given in column (3) of Table V. If, as in the previous numerical example, it is found that the level of projected migration is 0.42 percent a year $\left[= \frac{42,601}{5(2,024,711)}\right]$ the estimator will have the choice of either neglecting migration completely (i.e., assume the 1965 actual population to correspond to that in column (4) of Worksheet 6,) or using the percentages of the closest level range, in this case, column (2), Table V.

TABLE V
MODEL RELATIVE AGE DISTRIBUTION
OF MIGRATION BY MIGRATION LEVELS 2/

Age Group (end of period)	Level 0.5 - 1.0	1.0 - 1.5	1.5 - 2.5	2.5 - 3.0
(1)	(2)	(3)	(4)	(5)
Less than 15	20.8	19.5	23.3	24.3
15 through 24	24.1	27.0	22.2	22.7
25 " 44	50.3	46.4	45.7	43.2
45 " 64	2.9	6.3	7.7	8.6
65 and over	1.9	1.8	1.1	1.2
Total	100.0	100.0	100.0	100.0

1/ These percentages were found in an interstate migration study of the Negro population in the United States over the decades of the 1910's, 1920's, 1930's and 1940's. The study was prepared by the author, assisted by Mr. John F. Smith. It was supervised by Dr. E. J. Eberling, Chief Research and Statistics Section, Tennessee Department of Employment Security. Among other things, it was found that the average and standard deviations of these percentages are not only relatively small but become smaller the higher the level of migration.

2/ Migration levels refer to the percent that average yearly migration over a given period is to the population at the beginning of the period.

Two things will have to be noted here: First, the age brackets of Table V are not of five-year intervals as in Worksheet 6. [1] The use of the corresponding percentages will, therefore, result in estimates of the magnitudes of migration by age groups larger than those of Worksheet 6. It will, therefore, be necessary to combine age groups in column (4) of Worksheet 6 to correspond to the age breakdowns of estimated migration and the final population estimate expressed in terms of the age brackets of Table V. For example, the total of the first three line items of column (4) of Worksheet 6 will be taken as the 1965 population "Less than 15 years" of age (excluding migration); rows four and five as "15 through 24" and so on. [2]

It should be noted further that Table V does not indicate the sex breakdown of the model relative age distribution of migration. The reason for this is that the distribution of migration between the sexes is likely to depend, to a large extent, on institutional factors and, consequently, vary noticeably from one area of the world to another. In the United States total net migration for practically every State was found to be distributed fairly evenly between the two sexes so that if each percentage in Table V is reduced by one half, the resulting table will indicate the approximate age distribution of migration for either the males or the females. The estimator will have to make a choice as to how to break down these percentages between the two sexes on the basis of his knowledge of the conditions in the area under study.

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[1] The main reason for this grouping is to reduce the effects of random fluctuations.

[2] If the 1965 estimate is nevertheless required by five-year age brackets, it may be advisable, at the risk of significant inaccuracy, to distribute the estimated migration of the larger age brackets evenly among its five-year components.

PART II

CHAPTER IV

SHORT-CUT METHOD FOR PROJECTING POPULATION BY AGE AND SEX

In many instances, the migration assumption underlying component population projections is that the age-sex specific migration rates of the recent past may be assumed to continue over the projection period. [1] Furthermore, in many instances, current fertility and mortality rates may also be assumed to continue over the projection period. [2] If these assumptions are deemed acceptable for the purpose of a population projection, computations become not only much simpler to carry out but may be effected without knowledge of age specific fertility, mortality and migration rates. Obviously, such a simplification is most useful in underdeveloped countries where birth, death and migration records may either be totally absent or hopelessly deficient.

This short-cut method is described in Worksheet 10 below. The method requires only a knowledge of the age-sex distribution of the population at two previous censuses. We know that, for each age group in the first census to move up to the appropriate age group in the second census, it must be discounted by mortality and migration. The combined "discount rate" may thus be obtained by dividing the given age group at the second census by the corresponding age group at the first census. For example, suppose, as in Worksheet 10, that the two censuses were taken in 1940 and 1950; the combined discount rate for the 1940 male age group "20 through 24" may be obtained by dividing the 1950 male age group, "30 through 34" by the 1940 "20 through 24" thus obtaining 0.5950. It could then be said that if mortality and migration conditions of the age group "20 through 24" in the ten years following 1950 are similar to these conditions over the past ten years, then 59.50 percent of the 1950 "20 through 24" male age group will survive and stay in the country or area to become "30 through 34" in 1960. Hence by multiplying the "20 through 24" male age group of 1950 by 0.5950, an estimate [1] of the "30 through 34" age group in 1960 is obtained. [2]

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[1] Such as in Column 4, Worksheet 7 and in Column 9 of Worksheet 8 and 9 in the previous chapter.

[2] Especially where the projection period is relatively short.

Note that the same result would be obtained if one uses the survival ratios of 1940-50 in discounting the 1950 age groups and then make allowance for migration using the 1940-50 age-specific migration ratios.

The main disadvantage of this method is that, while it projects population by age, sex or any other breakdown, it reveals neither the expected age distribution of migration nor the expected magnitudes of migration and natural increase.

It must be noted, however, that the expected population change, ΔP, is readily available from the results of this method, so that if natural increase is independently determined, for example, by applying current rates found in another area that is believed to have similar demographic conditions, the level of migration may be then estimated as a residue. At any rate, when migration is substantial, its direction may be determined in a recent life table of a country believed to have similar mortality conditions. If the discount ratios are definitely higher than the life table survival ratios, in-migration is indicated. If they are substantially lower out-migration is indicated.

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[1] Errors in the two census counts of the base period may affect this estimate.

[2] These ratios are sometimes termed "migration - survival" ratios.

WORKSHEET 10
SHORT - CUT PROJECTION OF THE MALE POPULATION, BY AGE, 1950 - 1960

Age Group (1)	Population April 1940 (2)	Population April 1950 (3)	Disc. Ratios 1940-50 (4)	Population April 1960 (5)
Less than 5	260,494	239,511	0.6626	201,377
5 through 9	249,612	195,380	0.6021	164,572
10 " 14	227,028	172,596	0.5947	158,700
15 " 19	168,402	150,294	0.6508	117,638
20 " 24	142,411	135,014	0.5950	102,643
25 " 29	119,572	109,588	0.6560	97,811
30 " 34	101,527	84,738	0.5619	80,333
35 " 39	94,343	78,442	0.5646	71,890
40 " 44	79,391	57,043	0.5261	47,614
45 " 49	63,045	53,262	0.4910	44,288
50 " 54	44,072	41,767	0.6314	30,010
55 " 59	33,335	30,955	0.4382	26,152
60 " 64	33,961	27,825	0.2402	26,372
65 " 69)	- 41,374	34,360	0.2802	13,564
70 " 74)				6,684
75 and over)				9,628
TOTAL	1,658,567	1,410,775		1,199,276

WORKSHEET 11
SHORT - CUT PROJECTION OF THE FEMALE POPULATION, BY AGE, 1950 - 1960

Age Group (1)	Population April 1940 (2)	Population April 1950 (3)	Disc. Ratios 1940-50 (4)	Population April 1960 (5)
Less than 5	253, 434	230, 271	.6170	193, 490
5 through 9	236, 567	185, 497	.6608	156, 159
10 " 14	203, 879	156, 362	.6951	142, 077
15 " 19	174, 049	156, 319	.6311	122, 576
20 " 24	134, 530	141, 711	.5833	108, 687
25 " 29	126, 912	109, 844	.6127	98, 653
30 " 34	104, 035	78, 469	.5851	82, 660
35 " 39	94, 933	77, 753	.5411	67, 301
40 " 44	77, 319	60, 875	.5582	45, 912
45 " 49	59, 511	51, 369	.4634	42, 072
50 " 54	45, 786	43, 157	.5765	33, 980
55 " 59	32, 936	27, 576	.3825	23, 804
60 " 64	33, 532	26, 395	.2656	24, 880
65 " 69				10, 548
70 " 74	44, 029	34, 495	.2950	7, 011
75 and over				10, 176
TOTAL	1,621, 452	1,380, 093		1,169, 986

Instructions for Completing Worksheets 10 and 11: Short-cut Projection of the Male (and Female) Population, by Age.

Worksheets 10 and 11 illustrate the short-cut method described in the previous pages. In Worksheet 10, the male population is projected, by age, to 1960 on the basis of the 1940 and 1950 population counts. In Worksheet 11, the female population is similarly projected. The computations involved are, therefore, identical.

Column (1): Divide population into appropriate age brackets. (In this case, five year brackets are used but where the projection period is 10 years, the age groups may be listed in 10-year brackets).

Column (2): Opposite each age group in column (1), list the number reported in the first census (in this case, 1940).

Column (3): Opposite each age group in column (1), list the number reported in the second census (in this case, 1950).

Column (4): Divide each number in column (3) by the number reported for the age group 10 years younger in column (2) and place the result in the same row as the column (2) divisor. For example, for the 1950 "10 through 14 age group, divide their number (172, 596) by the number reported in 1940 as "less than 5" (260, 494) thus obtaining 0.6626 which is now listed in column (4) in the row corresponding to "less than 5." This "discount ratio," 0.6626, means that, neglecting census miscounts, 66.26 percent of those "less than 5" in 1940 survived and remained to 1950, i.e., until they reached the ages "10 through 14."

Column (5): On the assumption that, age-specific mortality and migration rates are the same as between 1940-50 and 1950-60 periods, the ratios obtained in column (4) should apply to the 1950-60 period. Hence, for each age group in 1950 (i.e., row in column (3) multiply by the discount ratio in the same row of column (4) and place the result two rows lower in column (5) indicating the age of the cohort at the end of the period, i. e., in 1960. For example, it was found that 66. 26 percent of the 1940 "less than 5" group survived and remained until 1950 to reach the "10 through 14" age group. On the basis of the constant mortality and migration assumption, it may be estimated that, again, 66. 26 percent of the 1950 "less than 5" (239, 511) survive and remain to 1960 to become "10 through 14. " Hence, the estimated 1960 "10 through 14" male should amount to 158, 700 persons.

It will be noted that the first two rows in column (5) will so far be left empty. But, on the assumption that fertility rates will also remain fairly the same 1/, the proportion of children in the first two age groups to the females of child-bearing ages (say, 15 through 45) will remain roughly the same as between 1950 and 1960. For instance, in 1950 it is found that the ratio of males "less than 5" to females "15 through 44" is 38. 3 percent. It is assumed that, in 1960, this ratio will remain the same and hence that the "less than 5" will be 38. 3 percent of 525, 789 or 201, 377. The same is done for the age group 5 through 9.

1/ This assumption is not absolutely essential for this method. If past fertility rates are otherwise determined they may be appropriately projected and the results discounted by appropriate (even if rough) survival ratios to obtain the first two age groups in 1960.

PART II
CHAPTER V

ESTIMATING AND PROJECTING THE NUMBER OF PERSONS REACHING A GIVEN AGE ANNUALLY

The component methods of population projection described in this handbook yield estimates of population by sex for five-year age groups at intervals of five or ten years. However, it may be required of the analyst to estimate and project a given single year of age for several years over a given period of time. For instance, the proper authorities may want to know the number of persons reaching the age of eligibility for voting at a past election and at future elections. Or it may be required to know, approximately, the number of persons reaching the age of compulsory school attendance, entry into college or the labor force, or compulsory military service.

For years when either a census count is published or reliable estimates or projections are made by five year age groups, 1/ an approximation of the number of persons of any given year of age may easily be obtained by application of a simple "formula" to the relevant five year group. For the interim years, a slightly more complicated method is necessary. In this chapter, a method for each of these two situations will be described.

Estimating the Size of a Single Year of Age From a Five Year Age Group

"A simple method yielding generally adequate results is based on a graduation formula devised by Sprague from which certain coefficients are deduced by Glover. 2/ These coefficients, or "Sprague multipliers", are used for the

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1/ Note that in many censuses, counts of single years of age are given for the younger and more relevant groups.

2/ "The method is described in A. J. Jaffe, Handbook of Statitical Methods for Demographers, op. cit., pp. 94 - 96. Reference is made to Glover's derivation from the Sprague formula as put forth by J. W. Glover in U.S. Bureau of the Census, United States Life Tables, Washington, 1921, pp. 344 and 345."

derivation of numbers in single-year ages within a given five-year age group from numbers in this and certain adjacent five-year age groups." 1/

It should first be noted that the reliability of this method is reduced if there is reason to believe that births, during the years of birth of the cohort that contains the single year of age in question, are definitely out of line with those of the neighboring years, or that the members of that single year of age experienced mortality or migration rates significantly different from the other members of the five-year age group. In such instances, it may be advisable to use a more elaborate technique which calls for reliable data of past births, and of survival ratios and migration rates by single years of age. Unfortunately, however, most underdeveloped countries, for which this handbook is devised, lack such data. The method using the Sprague multipliers may, therefore, prove to be the only practical alternative.

When using the Sprague multipliers for estimating the number of persons of a given age from an open-end listing of five-year age groups, the breaking down of the numbers in the last two age groups to their single year components presents some difficulties. 2/ Therefore, for clarity of presentation, primary emphasis will be given to estimates excluding the last two age groups in open end listings.

The relevant Sprague multipliers are given in Table VI below in three "panels". The first panel, entitled First End-Panel, applies to the age group "Less than 5". The second panel, headed First Next-to-End Panel, applies to the age group "5 through 9". The last panel, entitled Mid-Panel, 3/

1/ United Nations, Methods of Population Projections by Age and Sex, op. cit, p. 68.

2/ For these difficulties and how to solve them, see A. J. Jaffe's treatment of the Sprague Multipliers in Handbook of Statistical Methods for Demographers, op. cit., pp. 94-96.

3/ This is called Mid-Panel because it falls in the middle of the five panels in the complete table of the Sprague Multipliers.

is used in conjunction with all the remaining five-year age groups except the last two.

TABLE VI

PARTIAL LISTING OF THE SPRAGUE MULTIPLIERS

	N_1	N_2	N_3	N_4	N_5
First End - Panel					
n_1 ---	+ 0.3616	- 0.2768	+ 0.1488	- 0.0336	--
n_2 ---	+ 0.2640	- 0.0960	+ 0.0400	- 0.0080	--
n_3 ---	+ 0.1840	+ 0.0400	- 0.0320	+ 0.0080	--
n_4 ---	+ 0.1200	+ 0.1360	- 0.0720	+ 0.0160	--
n_5 ---	+ 0.0704	+ 0.1968	- 0.0848	+ 0.0176	--
First Next-to-End Panel					
n_1 ---	+ 0.0336	+ 0.2272	- 0.0752	+ 0.0144	--
n_2 ---	+ 0.0080	+ 0.2320	- 0.0480	+ 0.0080	--
n_3 ---	- 0.0080	+ 0.2160	- 0.0080	+ 0.0000	--
n_4 ---	- 0.0160	+ 0.1840	+ 0.0400	- 0.0080	--
n_5 ---	- 0.0176	+ 0.1408	+ 0.0912	- 0.0144	--
Mid - Panel					
n_1 ---	- 0.0128	+ 0.0848	+ 0.1504	- 0.0240	+ 0.0016
n_2 ---	- 0.0016	+ 0.0144	+ 0.2224	- 0.0416	+ 0.0064
n_3 ---	+ 0.0064	- 0.0336	+ 0.2544	- 0.0336	+ 0.0064
n_4 ---	+ 0.0064	- 0.0416	+ 0.2224	+ 0.0144	- 0.0016
n_5 ---	+ 0.0016	- 0.0240	+ 0.1504	+ 0.0848	- 0.0128

For the first two five-year age groups, N_1 designates the numbers in the first age group (Less than 5), N_2 the numbers in the second age group (5 through 9), and so on. The numbers, in the five single years of age within the five-year age groups are designated by n_1, n_2, n_3, n_4 and n_5 respectively. For the five-year age group "Less than 5", for example, n_1 refers to the number "Less than 1", n_2 to the number one year old, n_3 to the number two years old and so on.

In order to estimate the number in a given single year of age, say four years, one needs to multiply N_1 through N_4, i.e., the numbers in the first four five-year age groups, by the coefficients of the proper row in the relevant panel, in this case the row corresponding to n_5 in the first panel, and then add the results.

As an illustration, suppose that the male population of the Dominican Republic in 1950 is accurately enumerated by five year age groups in that country's 1950 census, as shown in Table VII below. Suppose further that an estimate of male children four years of age and seven years of age are required for that year.

TABLE VII

MALE POPULATION ENUMERATED IN THE 1950 CENSUS OF THE DOMINICAN REPUBLIC

Age Group	Male Population
Less than 5	189,383
5 through 9	150,704
10 " 14	141,661
15 " 19	101,552
20 " 24	105,152
25 " 29	77,620
30 " 34	59,618
35 " 39	60,137
40 " 44	47,183
45 " 49	36,551
50 " 54	30,712
55 " 59	21,049
60 " 64	19,716
65 " 69	9,502
70 " 74	8,176
75 " 79	3,941
80 " 84	3,697
85 and over	3,711
Total Males	1,070,065

From Table VII the following values are obtained for N for the appropriate age groups:

$$\begin{aligned} N_1 &= 189,383 \\ N_2 &= 150,704 \\ N_3 &= 141,661 \\ N_4 &= 101,552 \end{aligned}$$

For the number of male children four years of age, the relevant panel in Table VI is the first and the relevant row in this panel is that of n_5. Therefore:

$$\begin{aligned} n_5 = {} & +0.0704\ (189,383) + 0.1968\ (150,704) - 0.0848\ (141,661) \\ & +0.0176\ (101,552) \end{aligned}$$

Hence:

$$n_5 = 13,333 + 29,659 - 12,013 + 1,787$$

or:

$$n_5 = 32,766$$

The estimated number of male children four years of age in the Dominican Republic in 1950 is, therefore, equal to 32,766.

For the number of male children seven years of age, the relevant panel is the second and the relevant row is that of n_3 (because age 7 is the third youngest year of age in the group "5 through 9"). The estimated number of these children is therefore:

$$\begin{aligned} n_3 = {} & -0.0080\ (189,383) + 0.2160\ (150,704) \\ & -0.0080\ (141,661) + 0.0000\ (101,552) \end{aligned}$$

or:

$$n_3 = -1,515 + 32,552 - 1,133 = 29,904$$

For all the remaining five-year age groups, except the last two, the procedure for estimating the number in a single year of age, is somewhat similar to the one just explained. For these, however, the third panel (entitled Mid-Panel) is the relevant one. Furthermore, the number in the five-year age group of which the relevant single year of age is a component is always designated by N_3. The numbers in the preceding two age groups are designated by N_2 and N_1 and those in the succeeding age groups by N_4 and N_5. For example, if the single year of age the size of which is to be estimated is twelve, the size of the relevant five-year age group, "10 through 14", is denoted by N_3, the numbers in the preceding age groups "5 through 9" and "Less than 5" are denoted by N_2 and N_1 respectively while the number in the succeeding age groups "15 through 19" and "20 through 24" are represented by N_4 and N_5 respectively. However, the numbers in the single years of age within a given five-year age group are still designated n_1, n_2 ... n_5 as before.

In order, for example, to estimate the number of male persons 21 years of age from the 1950 census of the Dominican Republic (Table VII), it is necessary to proceed as follows:

The relevant five-year age group is "20 through 24" and its size is, therefore, designated by N_3. It follows that:

N_1 = "10 through 14" = 141,661
N_2 = "15 " 19" = 101,552
N_3 = "20 " 24" = 105,152
N_4 = "25 " 29" = 77,620
N_5 = "30 " 34" = 59,618

Moreover, because the single year of age twenty-one is the second age in the group, its size is designated by n_2 which indicates the relevant row in the (third) panel of the Sprague Multipliers. Therefore:

$$\begin{aligned} n_2 = & -0.0016\ (141,661) + 0.0144\ (101,552) \\ & +0.2224\ (105,152) - 0.0416\ (77,620) \\ & +0.0064\ (59,618) \end{aligned}$$

or:

$$n_2 = -227 + 1,462 + 23,386 - 3,229 + 382 = 21,774$$

and the estimated number of males twenty one years of age in the Dominican Republic in 1950 is 21,774 persons.

The reader who is not acquainted with the techniques just described might find it advisable to practice by using them in estimating the number of males in the single years of age from 1 through 19 in the Dominican Republic in 1950. Table VIII below is devised to help check the results.

TABLE VIII

COMPUTATION OF NUMBERS IN SINGLE YEARS OF AGE FOR THE MALE POPULATION LESS THAN TWENTY YEARS OF AGE, DOMINICAN REPUBLIC, 1950

Age	$f(N_1)$	$f(N_2)$	$f(N_3)$	$f(N_4)$	$f(N_5)$	Sum
Less than 1	+68,481	-41,715	+21,079	-3,412	--	44,433
1	+49,997	-14,468	+ 5,666	- 812	--	40,383
2	+34,846	+ 6,028	- 4,533	+ 812	--	37,153
3	+22,726	+20,496	-10,200	+1,625	--	34,647
4	+13,333	+29,659	-12,013	+1,787	--	32,766
5	+ 6,363	+34,240	-10,653	+1,462	--	31,412
6	+ 1,515	+34,963	- 6,800	+ 812	--	30,490
7	- 1,515	+32,552	- 1,133	+ 0	--	29,904
8	- 3,030	+27,730	+ 5,666	- 812	--	29,554
9	- 3,333	+21,219	+12,919	-1,462	--	29,343
10	- 2,424	+12,780	+21,306	-2,437	+ 168	29,393
11	- 303	+ 2,170	+31,505	-4,225	+ 673	29,820
12	+ 1,212	- 5,064	+36,039	-3,412	+ 673	29,448
13	+ 1,212	- 6,269	+31,505	+1,462	- 168	27,742
14	+ 303	- 3,617	+21,306	+8,612	-1,346	25,258
15	- 1,929	+12,013	+15,273	-2,524	+ 124	22,957
16	- 241	+ 2,040	+22,585	-4,374	+ 497	20,507
17	+ 965	- 4,760	+25,835	-3,533	+ 497	19,004
18	+ 965	- 5,893	+22,585	+1,514	- 124	19,047
19	+ 241	- 3,400	+15,273	+8,917	- 994	20,037

Estimating the Size of a Single Year of Age for Inter-Censal Years[1]

The method described in the previous section generally yields reliable estimates of the size of a given single year of age for census years or for years when estimates of five-year age groups are reliable. For the interim years, however, a different method has to be used. [2] Nevertheless the estimates obtained by the former method may still be taken as "check points" for the estimates obtained for the interim years.

In its simplest form, the method recommended here is based on the same assumption as in Worksheet 12 where an attempt is made to determine the number of males reaching the age of five years in the Dominican Republic between 1950 and 1955. The logic of this method may be summarized as follows: The relevant data show that out of 27,020 live births in 1944 [3], 31,412 persons reached the age of five years in 1950 (Table VIII) [4] The assumption made in Worksheet 12 is that over the periods 1945-1951, 1946-1952 .... 1949-1955 mortality and net migration rates are similar to those of the initial period (1944-1950) for the first five years of age, so that the same proportion of male live births in 1944 reaching the age of five in 1950 is applicable to the male live births of 1945 through 1949 reaching the age of five in 1951 through 1955. This assumption will be dropped later. Under it, however, estimates of the number of children five years of age in 1951 through 1955 may be easily obtained. The proportion of 1944 live male births reaching the age of five years in 1950 is obtained by dividing the number of males five years of age in 1950 (31,412) by the number dividing the number of males five years of age in 1950 (31,412) by the number of male live births in 1944 (27,020) thus obtaining a "discount ratio" of 1.1625. Under the assumption of Worksheet 12, this ratio may be applied to the male live births of 1945, 1946 . . . 1949 to obtain estimates of the number of males five years of age in 1951, 1952 . . . 1955. 1/ The step by step computations may be summarized as follows:

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[1] The following techniques are also applicable to years between a census and an estimate or two estimates of population by age.
The method devised in this section requires the use of past birth data.

[2] Because the single years of major interest are of the younger groups, these birth data would not have to be for periods too far in the past. If, however, these data are not available, and if there is no reason to believe that they fluctuated significantly, straight line interpolation between two estimates of the size of the given single year of age at the two dates may be advisable.

[3] For the sake of simplicity the census is assumed to have been taken on January 1, 1950. If it were taken, say, in the middle of that year, relevant male live births would be those that occurred between July 1, 1944 and July 1, 1945.

[4] The fact that more people are aged six years in 1950 than born in 1944 indicates that, deficiencies in the basic data aside, the net migration rate over the period 1944-1950 for the relevant cohort was positive and of larger absolute values than the mortality rate for the same group over the same period.

1/ Besides simplicity, one of the main advantage of this method is that any deficiency in birth statistics does not affect its reliability except when the degree of completeness of birth registration changes from year to year over the period of births (in this case 1944-1949). This is easy to see: The discount ratio (column (3), Worksheet 12) does not only reflect the balance between deaths and migration rate between 1944 and 1950, for the first five years of age, but also any deficiency in the statistics of births and of the relevant census counts. As long as these deficiencies remain the same over the relevant period, the relationship between births and the given age is not changed and the reliability of the estimates is not affected.

WORKSHEET 12

COMPUTING THE NUMBER OF MALE CHILDREN FIVE YEARS OF AGE IN THE DOMINICAN REPUBLIC 1950 - 1955

Year of Birth (1)	Male Live Births (2)	Discount Ratio (3)	Year of Age (4)	Estimated 5 year olds (5)
1944	27,020	1.1625	1950	31,411 1/
1945	30,417	1.1625	1951	35,360
1946	37,507	1.1625	1952	43,602
1947	34,712	1.1625	1953	40,353
1948	40,359	1.1625	1954	46,917
1949	42,921	1.1625	1955	49,896

1/ Not exactly equal to number given in the text because of rounding the discount ratio.

Instructions for Completing Worksheet 12: Computing the number of male children five years of age in the Dominican Republic, 1950-1955.

Column (1): List the relevant years of birth (1944-1949).

Column (2): Opposite each year of birth list actual male live births for that year.

Column (3): Divide the number of persons aged five years in 1950 by live births five years earlier and place this result in every row of this column.

Column (4): List the years for which the size of the single year of age is to be estimated.

Column (5): For each row, multiply column (2) by column (3).

The assumption that the discount ratio of the base period 1944-1950 is applicable need not be made unless there is reason to believe that it is realistic. When the number of births rises substantially after the birth year of the base period (in this case, 1944), a constant discount ratio does not only mean an increasing number of survivors to the given age but also an increasing number of migrants. To demonstrate this consider the rise in the number of male births between 1944 and 1946 in Worksheet 12. Assume for the sake of illustration that the death rate over the first five years of age is 0.03 or three percent and that this rate is applicable to the two periods 1944-1950 and 1946-1952. Assume further that the relevant statistics are accurate. In this case, and in the absence of migration for the given cohort, the discount ratio for the period 1944-1950 would be 0.9700. The fact that in-migration took place, however, raised that ratio to 1.1625. The migration rate for that group of births in the given period is, therefore, 1.1625 - 0.9700 = 0.1925, that is, an in-migration rate of 0.1925. In absolute quantities this in-migration amounted to 27,020 x 0.1925 = 5,201. By the same process of reasoning we find that for the second period under consideration, namely 1946-1952 the number of net in-migrants to this cohort is 37,507 x 0.1925 = 7,220. In other words, keeping the discount rate constant, the rise in the number of births increased the estimated number of immigrants from 5,201 to 7,220 persons.

The fact that changes in the size of migration to an area is largely independent of births in that area, shows the deficiency of holding discount rates constant when the number of births over the relevant period changes substantially. This deficiency, however, may be largely or completely overcome by a slight change in the method. [1]

In the illustration (Worksheet 13) of the adjusted method it is assumed that the male population of the Dominican Republic was adequately projected from 1950 to 1955, that with the help of the Sprague multipliers the size of the age-group "5 through 9" in 1955 was broken down to its single year components, and that the number of male children five years of age in 1955 was found to be 47,207. In this case, discount ratios for two different periods, namely 1944-1950 and 1949-1955 become easily available and may be used as controlling factors for the four interim periods, namely 1945-1951, 1946-1952, 1947-1953, and 1948-1954. The ratios for these four interim periods may now be obtained by interpolating between the ratios of the two limiting periods.

---

[1] This refinement is also useful when there is reason to believe that migration rates have changed noticeably over the relevant periods.

In our example, and under the assumptions just made, the discount ratio for the period 1944-1950 is still 1.1625 as in Worksheet 12. The ratio applicable to the period 1949-1955 may again be obtained by dividing the number of male children aged five years in 1955 (47,207) by the number of male births in 1949 (42,921), this ratio being 1.1000. By interpolation the discount ratios applicable to the 1945, 1946, 1947 and 1948 births may be obtained thusly:

For 1945: $R_{45} = 1.1625 + 1/5\,(1.1000 - 1.1625) = 1.1500$
For 1946: $R_{46} = 1.1625 + 2/5\,(1.1000 - 1.1625) = 1.1375$
For 1947: $R_{47} = 1.1625 + 3/5\,(1.1000 - 1.1625) = 1.1250$
For 1948: $R_{48} = 1.1625 + 4/5\,(1.1000 - 1.1625) = 1.1125$

The remaining steps in this method are the same as in the less refined one. 1/

1/ Compare Worksheet 12 with Worksheet 13.

WORKSHEET 13

COMPUTING THE NUMBER OF MALE CHILDREN FIVE YEARS OF AGE (UNDER GIVEN ASSUMPTIONS) IN THE DOMINICAN REPUBLIC 1950 - 1955

Year of Birth (1)	Male Live Births (2)	Discount Ratio (3)	Year of Age (4)	Estimated 5 years olds (5)
1944	27,020	1.1625	1950	31,411 1/
1945	30,417	1.1500	1951	34,980
1946	37,507	1.1375	1952	42,664
1947	34,712	1.1250	1953	39,051
1948	40,359	1.1125	1954	44,899
1949	42,921	1.1000	1955	47,213 1/

1/ Not exactly equal to number given in the text because of rounding the discount ratio.

PART II
CHAPTER VI

A. HALVING OF TEN - YEAR AGE GROUPS

It was stated in Part II, Chapters II and III, that it is much more convenient and adequate to state the age groups of a population to coincide with the estimate or projection period. If this period is set at five years while the Census break-down of the population is reported in ten year age brackets, one of the following two methods may prove helpful in reducing the reported ten year age groups to their five year constituents:

First Method for Halving Ten Year Age Groups

In this method, the statistician is required to go back to the original span of ten years during which the members of the given ten year age group were born and find the proportion of total live births that occurred in the first five years and the proportion that occurred in the second five years. These ratios may then be adjusted for the fact that the older half of the reported ten year age group lived, on the average, five years longer than the younger age group and, hence, had a slightly higher mortality rate.

To illustrate this procedure, suppose that the following data are found in the 1960 Census, and past vital statistics data, of a given country:

(1) Number "15 through 24" on January 1, 1960 = 72, 434;

(2) Total number of live births between January 1, 1935 and January 1, 1945 = 80, 306;

(3) Number of live births between January 1, 1935 and January 1, 1940 = 40, 956;

(4) Number of live births between January 1, 1940 and January 1, 1945 = 39, 350; and

(5) The five year mortality rate, of the age group "15 through 19" in 1960, around the year 1940 = 0. 02.

Suppose, further, that the 1960 "15 through 24" age group is to be broken into its two constituent five year age groups.

Step I Find what percentages (3) and (4) are of (2):

$$(3) = 0.51 \text{ of } (2) = B_1$$

$$(4) = 0.49 \text{ of } (2) = B_2$$

Step II If these percentages are applied to the 1960 count of the "15 through 24" to obtain the two component 5-year age groups, it would be assumed that a proportional number of people from each of these two groups have died since birth. This, of course, may be too strong an assumption. Even if the age-specific mortality rates were comparable for the two groups (a somewhat justifiable assumption); more would be expected to die from the 1960 age group "20 through 24" than from the age group "15 through 19" because the former group lived five years longer, on the average. In other words, while after 15 years of life for both groups the relationship given at birth may still be assumed to be valid, it may not be so assumed in 1960 when one group had already lived, on the average, five extra years. If the per cent of total births during the first five years of the ten year group 1935-1945 (hence, "20 through 24" in 1960) is denoted by B_1, while the per cent born during the second five years of that period (hence, "15 through 19" in 1960) is denoted by B_2, the relationship between B_1 and B_2 will have to change if it is to reflect the reduction of the older group by 2 per cent in the five extra years lived. Hence, the proportions reflecting the 1960 relationship between the two groups become:

$$nB_1 = \frac{0.98\ B_1}{B_2 + 0.98\ B_1} \quad \text{and} \quad nB_2 = \frac{B_2}{B_2 + 0.98\ B_1}$$

In our example:

$$nB_1 = 0.5049$$

$$nB_2 = 0.4951$$

Obviously, once one of the these two percentages is determined, the second percentage may be computed by subtracting the first from 1.0000 or 100 percent.

Step III: Once the relationship at birth between the two groups has been adjusted to the fact that one group lived five years longer on the average than the other (Step II), the new ratios, nB_1 and nB_2 may now be applied to the 1960 "15 through 24" age group to obtain estimates of the age groups "15 through 19" and "20 through 24". In our example:

"15 through 19" = 0.5049 x 72,434 = 36,572 and

"20 through 24" = 0.4951 x 72,434 = 35,862

Before describing the second method of halving ten year age groups, a few remarks concerning the first method are in order.

First, it must be obvious that the importance of Step II increases the higher the age group one intends to break down. The reason for this is that the mortality rate of the last five years of life becomes larger as the group involved becomes older.

Second, a greater degree of refinement may be obtained if it is taken into consideration, again in Step II, that the older age group was exposed to five extra years of migration in addition to the five extra years of mortality. If migration is thought to be substantial for the given age group, further adjustment will have to be made to take it into account. One way of obtaining a rough estimate of the allowance for migration is to go back to two recent Censuses and apply the methods described in Part II, Chapter II. In following this method, an estimate is made of the migration rate for the age group "15 through 24" in the last ten years. To estimate the rate of migration for the five-year age group "20 through 24" in the last five years of life, divide the initial estimate by four. [1] Once the allowance for migration is estimated, a step similar to Step II should be added to adjust the five-year age group estimates for mortality.

---

[1] A division by two will constitute an estimate of either migration of a five-year group over ten years or the migration of a ten-year group over five years.

The major shortcoming of the procedure is that in most underdeveloped countries, vital statistics data are not available for periods too distant in the past. 1/ This method will, hence, find wide use for the younger age groups while it might be of no practical use for the older groups.

Second Method for Halving Ten-Year Age Groups

This method is based on the assumption that a certain degree of smoothness exists in the age distribution data of the study area. It, however, applies to all age groups and, hence, does not suffer from the same shortcoming of the previous method.

The formula used in effecting the breakdown is the following Newton's formula:

$$f_{na} = 1/2 \left[f_n + 1/8 \ (f_n - 1 - f_n + 1) \right]$$

Where:

f_{na} = the number of persons in the first half of the given age group,

f_n = the total number reported in the ten-year age group, and

f_n-1 and f_n+1 = the number in the preceding and following ten-year age groups respectively.

To illustrate this, an example given in a United Nations Manual will be used. 2/ "Consider the official mid-year 1953 estimate of the Pacific Island Trust Territory, which was given by sex and age as follows:

1/ The fact that registration of live births may be incomplete is irrelevant to step I, unless there is reason to believe that the completeness of registration changed drastically as between the two five year periods over which initial births took place.

2/ United Nations, Manuals on Methods of Estimating Population, Manual III, Methods for Population Projections by Sex and Age, ST/SOA/Series A, Population studies, No. 25.

TABLE IX
(Unnumbered table in original text)

Age (Years)	Population Male	Population Female
TOTAL	25, 956	24, 929
0-4	4, 640	4, 310
5-9	2, 245	2, 148
10-14	2, 093	1, 978
15-19	2, 015	2, 037
20-24	2, 025	2, 026
25-34	3, 233	3, 124
35-44	2, 651	2, 637
45-54	2, 209	2, 155
55-64	1, 758	1, 605
65-74	1, 126	1, 024
75 and over	606	521
Age unknown	1, 355	1, 364

"For the number of males aged 25-29, Newton's formula gives:

$$f_{na} = 1/2\ [3,233 + 1/8\ (4,040 - 2,651)] = 1,703$$

"The number of males age 30-34 is obtained by subtracting 1, 703 from 3, 233, i.e., 1, 530.

"This method makes it possible to construct the estimates for five-year age groups up to age 64. To halve the 65-74 group, estimates for age groups over 75 years of age are required. These can be obtained by reference to Table IX. In this example, 2.46 per cent of males and 2.21 per cent of females of known age are ages 75 years and over. By interpolation, about

1.48 per cent of males should be aged 75-79, 0.71 per cent 80-84, and 0.27 per cent 85 and over; 1.36 per cent of females should be aged 75-79, 0.63 per cent 80-84, and 0.23 per cent 85 and over. The corresponding numbers are:

TABLE X
(Unnumbered table in original text)

Age (Years)	Males	Females
75-79	364	320
80-84	175	148
85 and over	66	54

"It is now possible to enter an estimate for the 75-84 group in Newton's formula and thus to halve the 65-74 group. The following results are obtained for the five-year age groups from 25 to 74; [1]

TABLE XI
(Unnumbered table in original text)

Age (Years)	Males	Females
25-29	1,703	1,651
30-34	1,530	1,473
35-39	1,390	1,379
40-44	1,261	1,258
45-49	1,160	1,142
50-54	1,049	1,013
55-59	947	873
60-64	811	732
65-69	639	583
70-74	487	441

---

[1] "In the application of formulas using several successive age groups, one must bear in mind that such formulas cannot be used where there have been past irregularities in the annual numbers of births. The number of the people affected by these irregularities have to be replaced by proper numbers before applying the formulas."

"In view of the large number of persons whose ages were not reported, it is important in this case to take account of any clues which may be available concerning possible peculiarities of the age distribution of this group. In case it can be assumed that their age distribution was similar to that of the remainder of the population, the estimates for all age groups should be 'pro-rated' to the total number of each sex." 1/

1/ "Pro-rating" means to distribute the number of persons of unknown ages proportionately among the different age groups. For example, the size of the male age group "0-4" (4, 640) is approximately 0.179 or 17.9 per cent of total males (25, 956). Its share will, hence, be 17.9 per cent of males of unknown ages, i.e., 0.179 x 1.355 = 243. The pro-rated size of that age group is, therefore, 4640 + 243 = 4883.

PART II
CHAPTER VI

B. PROJECTION OF BIRTHS

In order to project births into the future, three general steps may be necessary. First, past and current birth rates have to be determined or estimated; second, these birth rates have to be projected into the future on the basis of certain specific assumptions; and third, the projected rates have to be applied to a base population to obtain the projected magnitudes.

Determining Past and Current Birth Rates

It should be noted at the outset that there are at least two major concepts of the birth rate--the crude and the sex-age adjusted rates. Roughly speaking, the crude birth rate is obtained by dividing the number of live births during a year by the total number of people at the beginning (or, better, at the middle) of that year. If the actual number of live births is denoted by B, and the base population by P_b; the crude birth rate R_c, may be obtained by direct substitution in the following equation:

$$R_c = \frac{B}{P_b}$$

The sex-age adjusted birth rate not only involves a more difficult technique but it also presents a more refined approach to the projection of births. In general, the sex-age adjusted birth rate relates the number of live births not to the total population but to a segment of it, usually the female population of child-bearing ages. The fact that this rate is more refined and more relevant to the projection of fertility levels than the crude rate should be apparent, for it relates the number of births to a segment of the population that is directly involved in the birth process rather than to the total population which contains children and older people whose numbers have no direct effect on the number of births. Futhermore, the sex-age adjusted birth rate usually fluctuates less from year to year than the crude birth rate which makes it a more reliable variable for projection purposes. However, the difference between the results of the application of the two rates is usually not so large over time and the sex-age adjusted birth rate may be recommended only when the age distribution of the base population shows a fluctuating degree of smoothness between counts.

When either rate is computed for one year--i.e., based on a single year's live births--it might contain undesirable random fluctuations. It is better, therefore, to take as the relevant year's births the average live births of three or five years centered around it. As an illustration, let us suppose that it is desired to compute the average number of live births around the year 1956 (a census year) in the Fiji Islands. In 1956, the reported number of live births was given at 7,205 males and 6,871 females, a total of 14,076. Instead of using this figure in computing the birth rate in 1956, it may be desirable to use the average of the three years centering around 1956. In 1955 the total number of live births was given at 13,067 and in 1957 at 14,845. Following this procedure the average number of live births over the period 1955 - 1957 is $\frac{13,067 + 14,076 + 14,845}{3} = 13,996$.

In order to compute the crude birth rate (R_c) for the Fiji Islands in 1956, it is necessary to have the total number of persons living there in that year. This is the reason why the birth rate is usually computed for a census year. In 1956, the Fiji Island census of population set the total population at 345,737 persons. The average number of births around that year was 13,996 and, the crude birth rate $R_c = \frac{B}{P_b}$

or:

$$R_c = \frac{13,996}{345,737} = 0.0405$$

It may be desirable, however, to obtain the crude birth rate for male and female births separately. In this case, the rate for each sex is obtained by dividing male and female average births respectively by total population. To obtain the male birth rate, for example, the average births in the years 1955, 1956, and 1957 are first computed. These, it will be found, amount to 7,198 persons. 1/ The 1956 crude birth rate for male births is therefore:

$$ {}_{'56}R_c = \frac{7,198}{345,737} = 0.0208 $$

If it is desired to compute the usually more reliable sex-age adjusted birth rate, 2/ it is essential to secure first the number of women aged 15 through 44 years, i.e., of "child-bearing" ages. In the 1956 census of the Fiji Islands, this was reported as in Column (2) of Worksheet 14. In order to compute the sex-age adjusted birth rate, it is necessary to weight the respective five-year age groups of the female population by a set of weights that reflect the comparative fertilities of these groups. The weights in Column (3) of Worksheet 14 were chosen not only because they reflect this relationship "but also [because] the sum of their products with the corresponding numbers of women in the various age groups [is] ordinarily ... of the same order of magnitude as the total population." 3/ In this way, the sex-age adjusted birth rate will be of the same order of magnitude as the crude birth rate.

1/ Male births in 1955, 1956, and 1957 were reported at 6,741, 7,205 and 7,649 respectively.

2/ These computations are explained in more detail in, United Nations, Methods of Population Projections by Age and Sex. ST/SOA/Series A, No. 25 pp. 41 - 44.

3/ Ibid, p-42.

Worksheet 14 describes the computation of the sex-age adjusted birth rate for male births in the Fiji Island in 1956. The reader will find it self-explanatory.

WORKSHEET 14

COMPUTATION OF SEX-AGE ADJUSTED BIRTH RATE, FIJI ISLANDS MALES, 1956

Age Group	Female Population 1956	Weight	Product	Average Births 1955-57 1/	Sex-Age Adjusted Birth Rate
(1)	(2)	(3)	(4):(2)x(3)	(5)	(6):(5) ÷ (4)
15 through 19	18, 162	1	18, 162	-	-
20 " 24	14, 776	7	103, 432	-	-
25 " 29	12, 105	7	84, 735	-	-
30 " 34	9, 757	6	58, 542	-	-
35 " 39	8, 749	4	34, 996	-	-
40 " 44	6, 775	1	6, 775	-	-
Total	-	-	306, 642	7, 198	0.0235

1/ Male births in 1955, 1956 and 1957 were reported at 6, 741, 7, 205 and 7, 649 respectively.

Projecting Birth Rates

If a series of past birth rates is obtained, it may be projected into the future by extrapolation after a choice is made as to the proper base period. If only the current birth rate may be determined adequately, it may be assumed to remain constant over a short period of time into the future or may be changed slightly by the estimator on the basis of his personal knowledge of the demographic conditions in the area. For the sake of illustration, both the adjusted and the crude birth rates for each sex in the Fiji Islands will be held constant until 1966.

Projecting the Number of Births

Once the birth rate for a future date is obtained, future births may be estimated by multiplying this rate by a base population for that date. When the crude birth rate is used, the relevant base population will be projected total population. For the sake of illustration, suppose that the Fiji Island's total population projected to 1961 amounted to 431, 836 persons. 1/ If the crude birth rate for male births is to be used, the estimated number of male births in 1966 may be obtained by multiplying the 1966 projected crude birth rate for male births (0.0208) by the projected total number of people in 1966. Hence:

$$_{'66}B = 0.0208 \times 431{,}836 = 8{,}982$$

When the sex-age adjusted birth rate is used, the relevant base population will be the projected female weighted population aged 15 through 44 at the end of the projection period. Using the short-cut method of Part II Chapter IV, the 1966 projections of females of child-bearing ages in the Fiji Islands are illustrated in Worksheet 15 and the projected male births in 1966, using the sex-age adjusted birth rate are estimated to be 10, 005.

1/ Projected arithmetically, 1946 - 56 base period.

WORKSHEET 15

COMPUTATIONS OF FUTURE SEX-AGE ADJUSTED BIRTHS, FIJI ISLANDS MALES, 1966

Age Group	Projected Female Population 1966	Weight	Product	Projected Male Birth Rate 1966 1/	Projected Male Births 1966
(1)	(2)	(3)	(4):(2)x(3)	(5)	(6):(4)x(5)
15 through 19	24,049	1	24,049	-	-
20 " 24	21,079	7	147,553	-	-
25 " 29	17,793	7	124,551	-	-
30 " 34	13,069	6	78,414	-	-
35 " 39	10,606	4	42,424	-	-
40 " 44	8,734	1	8,734	-	-
Total	-	-	425,725	0.0235	10,005

1/ Assumed to be the same as in 1956 (see text).

If estimates of the number of births are desired for the years between 1956 and 1966, a simple arithmetic interpolation can be applied to the 1956 and 1966 estimates as illustrated in Table XII.

TABLE XII

INTERPOLATED SEX-AGE ADJUSTED MALE BIRTHS, FIJI ISLANDS, 1956-1966

Year	Number of Births
1956	7,198 1/
1957	7,479
1958	7,760
1959	8,041
1960	8,322
1961	8,603
1962	8,884
1963	9,165
1964	9,446
1965	9,727
1966	10,005 2/

1/ From Worksheet 14, Column 5.

2/ From Worksheet 15, Column 6.

PART II
CHAPTER VI

C. COMPUTING SURVIVAL RATIOS

One of the main elements of a component method of computation is the choice of an appropriate set of age-sex survival ratios with which age-sex cohorts are discounted to obtain their probable numbers n years later. When using these survival ratios to obtain the number of survivors in each cohort from one census to another in order to determine the level and age distribution of migration, 1/ it is essential that these ratios be accurate. It may easily be shown that a discrepancy of one percentage point in an age-sex survival ratio will result in a much larger relative distortion in the corresponding estimate of migration of the given age-sex group and that it may, in some instances, reverse its sign. 2/ However, when age-sex cohorts are projected into the future with the use of survival ratios, 3/ the degree of accuracy of these ratios that is absolutely necessary for most purposes becomes relatively lower because the relative distortion in the projected size of the cohort is only comparable to the error in the given ratio.

Let us note at the outset that there are at least two different types of survival ratios--"life table survival ratios" and "census survival ratios". They differ conceptually but both may be taken as approximations of the true probability of survival. Their main difference stems from the fact that the life table survival ratios attempt to approximate the true probability of survival while the census survival ratios intentionally take into account the deficiencies in census statistics. 4/ In other words, while the life table

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1/ As in Chapter II, Part II above.

2/ A distortion of, say, one percentage point in the survival ratio has approximately the same effect on the resulting migration estimate as a one percent distortion in the reported size of the cohort at the first census. For an example of this latter effect see page 84 Chapter II, Part II.

3/ As in Chapter III, Part II.

4/ For a detailed discussion of the main differences between the two types of ratios see, E.S. Lee, et. al., Population Redistribution and Economic Growth, United States, 1870-1950, Vol. I, The American Philosophical Society, Philadelphia, 1957, pp. 15 - 27.

ratios find use in different fields of research (e.g. medical, demographic, actuarial, etc.), the census ratios are essentially a tool for demography.

Life Table Survival Ratios

A complete life table contains a number of functions beside the survival ratios. Some of these functions precede, and some succeed, these ratios in the sequence of computation of a life table. This instruction will be restricted to the former functions except for two additional functions that may be of general interest.

Furthermore, a so-called "complete"--or, as referred to here, "detailed"--life table presents ratios of survival for every year of age and is, therefore, generally more reliable than the "abridged" life tables which contain survival ratios for age groups of more than one year. Detailed life tables are not quite essential to the present handbook because the techniques outlined deal specifically with five-year age groups rather than single years of age. Therefore, only the techniques of constructing abridged life tables will be explained.

Functions Included in Life Table Survival Ratios: In the sequence of computations of the life table survival ratios we encounter the following functions:[1]

m_x = The death rate of persons of age x.

q_x = The probability of dying within a given age interval.

l_x = The number of survivors to a specified age from an assumed initial number of births.

L_x = The number of years lived collectively by those survivors within the given age interval.

P_x = The survival ratio from one age interval to another.

T_x = The collective number of years yet to be lived by the survivors to a given age.

e^o_x = The expectation of life at a given age.

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[1] As mentioned earlier, the last two functions are not needed for the computation of survival ratios but will be discussed because of their wide usefulness.

a) The m_x Function: This function gives the death rate of each age-sex group on or around a given date shown in the title of the life table. Worksheets 16 and 17 undertake the computation of m_x values for the males and females of the Fiji Islands. Because these worksheets are self-explanatory, the discussion of their construction will be restricted to a minimum.

WORKSHEET 16

AGE-SEX SPECIFIC DEATH RATES FOR FIJI ISLANDS MALES 1950-1952

	Deaths Reported in Group					Age-Sex Specific Death Rate m_x
Age Group	1950	1951	1952	Average Deaths 1950-52	Average Population 1951	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Less than 1	366	374	451	397	6,068	0.0654
1 through 4	205	175	280	220	20,939	0.0105
5 " 9	45	41	40	42	23,292	0.0018
10 " 14	32	23	22	26	19,237	0.0014
15 " 19	51	31	32	38	15,692	0.0024
20 " 24	32	41	37	37	12,955	0.0029
25 " 29	38	33	44	38	11,287	0.0034
30 " 34	43	34	36	38	9,063	0.0042
35 " 39	47	42	44	44	7,962	0.0055
40 " 44	55	48	39	47	6,352	0.0074
45 " 49	45	51	67	54	5,291	0.0102
50 " 54	75	65	90	77	4,223	0.0182
55 " 59	78	72	85	78	3,940	0.0198
60 " 64	183	167	203	184	4,547	0.0405
65 " 69	114	110	106	110	2,699	0.0408
70 " 74	123	114	198	145	1,829	0.0793
75 and over	233	227	217	226	1,911	0.1183

WORKSHEET 17

AGE-SEX SPECIFIC DEATH RATES FOR FIJI ISLANDS FEMALES, 1950-1952

Age Group	Deaths Reported in Group			Average Deaths 1950-52	Average Population 1951	Age-Sex Specific Death Rate m_x
	1950	1951	1952			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Less than 1	305	274	353	311	5,950	0.0523
1 through 4	173	206	243	207	20,432	0.0101
5 " 9	27	41	48	39	22,638	0.0017
10 " 14	31	32	34	32	18,407	0.0017
15 " 19	53	57	41	50	15,259	0.0033
20 " 24	66	52	65	61	12,904	0.0047
25 " 29	65	53	76	65	11,045	0.0059
30 " 34	51	65	51	56	8,663	0.0065
35 " 39	51	50	46	49	7,111	0.0069
40 " 44	47	41	50	46	5,607	0.0082
45 " 49	41	44	33	39	4,179	0.0093
50 " 54	59	40	53	51	3,714	0.0137
55 " 59	46	45	40	44	2,661	0.0165
60 " 64	124	96	127	116	2,704	0.0429
65 " 69	59	69	73	67	1,436	0.0467
70 " 74	106	129	107	114	1,056	0.1080
75 and over	141	130	145	139	1,025	0.1356

Instructions for Completing Worksheets 16 and 17: Computation of Age-Sex Specific Death Rates.

Column (1): List ages in brackets of five years. Because of the significantly uneven distribution of deaths between the first year of age and the next four year, list these two brackets separately in the first two rows respectively.

Column (2), (3), and (4): List opposite each age bracket the reported deaths (adjusted for incomplete registration, if necessary) for the age group over three years centering around a census year (in this case, 1951).

Column (5): For each row, average figures in columns (2), (3), and (4) to obtain average deaths per year around the central date.

Column (6): Opposite each age bracket list the corresponding 1951 census count (adjusted for census miscount if necessary).

Column (7): Divide column (5) by column (6), for each row, to obtain the age-sex specific death rates (m_x).

b) The q_x Function: There is no simple mathematical formula for the transformation of age-sex specific death rates (m_x) into "probability of death before reaching an older age-sex group" -- i.e., life table death rates q_x. Fortunately, however, L. J. Reed and Margaret Merrell have published conversion tables 1/ whereby the transformation of m_x into q_x may be simply accomplished. 2/

1/ In "A Short Method for Constructing an Abridged Life Table," The American Journal of Hygiene, Vol. 30, No. 2, September 1939. Reproduced in full in A. J. Jaffe, Op. cit., pp. 12-27.

2/ Relevant Reed and Merrell tables (q_o, $4q_1$, and $5q_x$) are reproduced below.

Appendix 2

Table I

Values of q_0 associated with m_0 by the equation:

$$q_0 = 1 - e^{-m_0(9539 - .5509\, m_0)}$$

m_0	q_0	Δ	m_0	q_0	Δ
		.000			.000
.000	.000 000	953	.050	.045 262	857
.001	.000 953	951	.051	.046 119	856
.002	.001 904	949	.052	.046 975	853
.003	.002 853	947	.053	.047 828	851
.004	.003 800	944	.054	.048 679	850
.005	.004 744	943	.055	.049 529	849
.006	.005 687	941	.056	.050 378	846
.007	.006 628	939	.057	.051 224	845
.008	.007 567	937	.058	.052 069	842
.009	.008 504	935	.059	.052 911	842
.010	.009 439	933	.060	.053 753	839
.011	.010 372	931	.061	.054 592	837
.012	.011 303	929	.062	.055 429	835
.013	.012 232	928	.063	.056 264	834
.014	.013 160	925	.064	.057 098	832
.015	.014 085	923	.065	.057 930	831
.016	.015 008	921	.066	.058 761	828
.017	.015 929	919	.067	.059 589	827
.018	.016 848	918	.068	.060 416	825
.019	.017 766	915	.069	.061 241	823
.020	.018 681	913	.070	.062 064	821
.021	.019 594	912	.071	.062 885	820
.022	.020 506	910	.072	.063 705	818
.023	.021 416	907	.073	.064 523	817
.024	.022 323	906	.074	.065 340	814
.025	.023 229	904	.075	.066 154	813
.026	.024 133	902	.076	.066 967	811
.027	.025 035	900	.077	.067 778	809
.028	.025 935	898	.078	.068 587	808
.029	.026 833	897	.079	.069 395	805
.030	.027 730	894	.080	.070 200	805
.031	.028 624	892	.081	.071 005	802
.032	.029 516	891	.082	.071 807	801
.033	.030 407	889	.083	.072 608	799
.034	.031 296	886	.084	.073 407	797
.035	.032 182	885	.085	.074 204	795
.036	.033 067	884	.086	.074 999	794
.037	.033 951	881	.087	.075 793	793
.038	.034 832	879	.088	.076 586	790
.039	.035 711	878	.089	.077 376	789
.040	.036 589	875	.090	.078 165	787
.041	.037 464	874	.091	.078 952	786
.042	.038 338	872	.092	.079 738	783
.043	.039 210	870	.093	.080 521	782
.044	.040 080	868	.094	.081 303	780
.045	.040 948	866	.095	.082 083	779
.046	.041 814	865	.096	.082 862	777
.047	.042 679	863	.097	.083 639	776
.048	.043 542	861	.098	.084 415	773
.049	.044 403	859	.099	.085 188	772
.050	.045 262	857	.100	.085 960	770

Table I - Continued

Values of q_0 associated with m_0 by the equation:

$$q_0 = 1 - e^{-m_0(.9539 - .5509\, m_0)}$$

m_0	q_0	Δ	m_0	q_0	Δ
		.000			.000
.100	.085 960	770	.150	.122 510	691
.101	.086 730	769	.151	.123 201	690
.102	.087 499	767	.152	.123 891	689
.103	.088 266	766	.153	.124 580	686
.104	.089 032	764	.154	.125 266	685
.105	.089 796	762	.155	.125 951	684
.106	.090 558	760	.156	.126 635	683
.107	.091 318	759	.157	.127 318	680
.108	.092 077	757	.158	.127 998	679
.109	.092 834	756	.159	.128 677	678
.110	.093 590	754	.160	.129 355	677
.111	.094 344	752	.161	.130 032	674
.112	.095 096	751	.162	.130 706	673
.113	.095 847	749	.163	.131 379	672
.114	.096 596	747	.164	.132 051	671
.115	.097 343	746	.165	.132 722	669
.116	.098 089	745	.166	.133 391	667
.117	.098 834	742	.167	.134 058	666
.118	.099 576	741	.168	.134 724	665
.119	.100 317	739	.169	.135 389	663
.120	.101 056	739	.170	.136 052	661
.121	.101 795	736	.171	.136 713	660
.122	.102 531	734	.172	.137 373	659
.123	.103 265	733	.173	.138 032	657
.124	.103 998	732	.174	.138 689	656
.125	.104 730	730	.175	.139 345	654
.126	.105 460	728	.176	.139 999	653
.127	.106 188	727	.177	.140 652	651
.128	.106 915	725	.178	.141 303	651
.129	.107 640	724	.179	.141 954	648
.130	.108 364	722	.180	.142 602	647
.131	.109 086	720	.181	.143 249	646
.132	.109 806	719	.182	.143 895	644
.133	.110 525	717	.183	.144 539	643
.134	.111 242	716	.184	.145 182	641
.135	.111 958	714	.185	.145 823	640
.136	.112 672	713	.186	.146 463	639
.137	.113 385	711	.187	.147 102	637
.138	.114 096	710	.188	.147 739	636
.139	.114 806	708	.189	.148 375	634
.140	.115 514	706	.190	.149 009	633
.141	.116 220	705	.191	.149 642	631
.142	.116 925	704	.192	.150 273	630
.143	.117 629	702	.193	.150 903	629
.144	.118 331	700	.194	.151 532	628
.145	.119 031	699	.195	.152 160	625
.146	.119 730	697	.196	.152 785	625
.147	.120 427	696	.197	.153 410	623
.148	.121 123	694	.198	.154 033	622
.149	.121 817	693	.199	.154 655	620
.150	.122 510	691	.200	.155 275	

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AN ABRIDGED LIFE TABLE

Table IV

Values of $_4q_1$ associated with $_4m_1$ by the equation:

$$_4q_1 = 1 - e^{-4\,{}_4m_1(.9806 - 2.079\,{}_4m_1)}$$

$_4m_1$	$_4q_1$	Δ	$_4m_1$	$_4q_1$	Δ
		.00			.00
.000	.000 000	3 906	.020	.072 370	3 316
.001	.003 906	3 875	.021	.075 686	3 289
.002	.007 781	3 843	.022	.078 975	3 262
.003	.011 624	3 812	.023	.082 237	3 235
.004	.015 436	3 781	.024	.085 472	3 209
.005	.019 217	3 750	.025	.088 681	3 183
.006	.022 967	3 720	.026	.091 864	3 156
.007	.026 687	3 689	.027	.095 020	3 131
.008	.030 376	3 659	.028	.098 151	3 104
.009	.034 035	3 630	.029	.101 255	3 079
.010	.037 665	3 600	.030	.104 334	3 054
.011	.041 265	3 570	.031	.107 388	3 028
.012	.044 835	3 542	.032	.110 416	3 003
.013	.048 377	3 512	.033	.113 419	2 979
.014	.051 889	3 484	.034	.116 398	2 954
.015	.055 373	3 455	.035	.119 352	2 929
.016	.058 828	3 427	.036	.122 281	2 905
.017	.062 255	3 399	.037	.125 186	2 882
.018	.065 654	3 372	.038	.12[illegible] 068	2 856
.019	.069 026	3 344	.039	.13[illegible] 924	2 834
.020	.072 370	3 316	.040	.133 758	

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AN ABRIDGED LIFE TABLE

Table V

Values of $_5q_x$ associated with $_5m_x$ by the equation:

$$_5q_x = 1 - e^{-5\,_5m_x - .008\,(5)^3\,_5m_x^2}$$

$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ
		.00			.00			.00
.000	.000 000	4 989	.050	.223 144	3 952	.100	.399 504	3 116
.001	.004 989	4 965	.051	.227 096	3 935	.101	.402 620	3 100
.002	.009 954	4 943	.052	.231 031	3 915	.102	.405 720	3 085
.003	.014 897	4 920	.053	.234 946	3 897	.103	.408 805	3 070
.004	.019 817	4 897	.054	.238 843	3 879	.104	.411 875	3 056
.005	.024 714	4 876	.055	.242 722	3 861	.105	.414 931	3 041
.006	.029 590	4 852	.056	.246 583	3 842	.106	.417 972	3 026
.007	.034 442	4 830	.057	.250 425	3 824	.107	.420 998	3 011
.008	.039 272	4 808	.058	.254 249	3 807	.108	.424 009	2 998
.009	.044 080	4 786	.059	.258 056	3 788	.109	.427 007	2 982
.010	.048 866	4 763	.060	.261 844	3 770	.110	.429 989	2 969
.011	.053 629	4 742	.061	.265 614	3 753	.111	.432 958	2 953
.012	.058 371	4 720	.062	.269 367	3 735	.112	.435 911	2 940
.013	.063 091	4 698	.063	.273 102	3 717	.113	.438 851	2 926
.014	.067 789	4 676	.064	.276 819	3 700	.114	.441 777	2 911
.015	.072 465	4 655	.065	.280 519	3 682	.115	.444 688	2 897
.016	.077 120	4 633	.066	.284 201	3 665	.116	.447 585	2 883
.017	.081 753	4 612	.067	.287 866	3 647	.117	.450 468	2 870
.018	.086 365	4 590	.068	.291 513	3 630	.118	.453 338	2 855
.019	.090 955	4 570	.069	.295 143	3 613	.119	.456 193	2 842
.020	.095 525	4 547	.070	.298 756	3 596	.120	.459 035	2 827
.021	.100 072	4 527	.071	.302 352	3 579	.121	.461 862	2 815
.022	.104 599	4 506	.072	.305 931	3 562	.122	.464 677	2 800
.023	.109 105	4 485	.073	.309 493	3 545	.123	.467 477	2 787
.024	.113 590	4 464	.074	.313 038	3 528	.124	.470 264	2 773
.025	.118 054	4 444	.075	.316 566	3 511	.125	.473 037	2 760
.026	.122 498	4 423	.076	.320 077	3 495	.126	.475 797	2 746
.027	.126 921	4 402	.077	.323 572	3 478	.127	.478 543	2 733
.028	.131 323	4 382	.078	.327 050	3 461	.128	.481 276	2 720
.029	.135 705	4 361	.079	.330 511	3 445	.129	.483 996	2 707
.030	.140 066	4 341	.080	.333 956	3 429	.130	.486 703	2 693
.031	.144 407	4 321	.081	.337 385	3 412	.131	.489 396	2 680
.032	.148 728	4 301	.082	.340 797	3 396	.132	.492 076	2 667
.033	.153 029	4 281	.083	.344 193	3 380	.133	.494 743	2 655
.034	.157 310	4 261	.084	.347 573	3 364	.134	.497 398	2 641
.035	.161 571	4 241	.085	.350 937	3 347	.135	.500 039	2 628
.036	.165 812	4 221	.086	.354 284	3 332	.136	.502 667	2 616
.037	.170 033	4 201	.087	.357 616	3 316	.137	.505 283	2 603
.038	.174 234	4 182	.088	.360 932	3 300	.138	.507 886	2 590
.039	.178 416	4 162	.089	.364 232	3 284	.139	.510 476	2 577
.040	.182 578	4 143	.090	.367 516	3 268	.140	.513 053	2 565
.041	.186 721	4 123	.091	.370 784	3 253	.141	.515 618	2 552
.042	.190 844	4 104	.092	.374 037	3 237	.142	.518 170	2 540
.043	.194 948	4 085	.093	.377 274	3 222	.143	.520 710	2 527
.044	.199 033	4 066	.094	.380 496	3 206	.144	.523 237	2 515
.045	.203 099	4 047	.095	.383 702	3 191	.145	.525 752	2 503
.046	.207 146	4 028	.096	.386 893	3 176	.146	.528 255	2 490
.047	.211 174	4 008	.097	.390 069	3 160	.147	.530 745	2 478
.048	.215 182	3 990	.098	.393 229	3 145	.148	.533 223	2 466
.049	.219 172	3 972	.099	.396 374	3 130	.149	.535 689	2 454
.050	.223 144	3 952	.100	.399 504	3 116	.150	.538 143	2 442

Table V - Continued

Values of $_5q_x$ associated with $_5m_x$ by the equation:

$$_5q_x = 1 - e^{-5\,{}_5m_x - .008\,(5)^3\,{}_5m_x^2}$$

$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ
		.00			.00			.00
.150	.538 143	2 442	.200	.646 545	1 904	.250	.730 854	1 476
.151	.540 585	2 430	.201	.648 449	1 894	.251	.732 330	1 469
.152	.543 015	2 418	.202	.650 343	1 885	.252	.733 799	1 462
.153	.545 433	2 406	.203	.652 228	1 876	.253	.735 261	1 453
.154	.547 839	2 394	.204	.654 104	1 866	.254	.736 714	1 447
.155	.550 233	2 382	.205	.655 970	1 856	.255	.738 161	1 439
.156	.552 615	2 371	.206	.657 826	1 847	.256	.739 600	1 432
.157	.554 986	2 359	.207	.659 673	1 838	.257	.741 032	1 424
.158	.557 345	2 347	.208	.661 511	1 829	.258	.742 456	1 417
.159	.559 692	2 336	.209	.663 340	1 819	.259	.743 873	1 409
.160	.562 028	2 324	.210	.665 159	1 810	.260	.745 282	1 403
.161	.564 352	2 313	.211	.666 969	1 802	.261	.746 685	1 395
.162	.566 665	2 301	.212	.668 771	1 792	.262	.748 080	1 388
.163	.568 966	2 290	.213	.670 563	1 783	.263	.749 468	1 381
.164	.571 256	2 279	.214	.672 346	1 774	.264	.750 849	1 374
.165	.573 535	2 267	.215	.674 120	1 765	.265	.752 223	1 366
.166	.575 802	2 257	.216	.675 885	1 756	.266	.753 589	1 360
.167	.578 059	2 245	.217	.677 641	1 747	.267	.754 949	1 353
.168	.580 304	2 234	.218	.679 388	1 739	.268	.756 302	1 345
.169	.582 538	2 223	.219	.681 127	1 729	.269	.757 647	1 339
.170	.584 761	2 211	.220	.682 856	1 721	.270	.758 986	1 332
.171	.586 972	2 201	.221	.684 577	1 712	.271	.760 318	1 325
.172	.589 173	2 190	.222	.686 289	1 704	.272	.761 643	1 318
.173	.591 363	2 180	.223	.687 993	1 695	.273	.762 961	1 311
.174	.593 543	2 168	.224	.689 688	1 686	.274	.764 272	1 304
.175	.595 711	2 157	.225	.691 374	1 678	.275	.765 576	1 298
.176	.597 868	2 147	.226	.693 052	1 669	.276	.766 874	1 291
.177	.600 015	2 137	.227	.694 721	1 661	.277	.768 165	1 284
.178	.602 152	2 125	.228	.696 382	1 652	.278	.769 449	1 278
.179	.604 277	2 115	.229	.698 034	1 644	.279	.770 727	1 271
.180	.606 392	2 105	.230	.699 678	1 636	.280	.771 998	1 264
.181	.608 497	2 094	.231	.701 314	1 627	.281	.773 262	1 258
.182	.610 591	2 083	.232	.702 941	1 619	.282	.774 520	1 251
.183	.612 674	2 073	.233	.704 560	1 611	.283	.775 771	1 245
.184	.614 747	2 063	.234	.706 171	1 602	.284	.777 016	1 239
.185	.616 810	2 053	.235	.707 773	1 595	.285	.778 255	1 231
.186	.618 863	2 042	.236	.709 368	1 586	.286	.779 486	1 226
.187	.620 905	2 032	.237	.710 954	1 578	.287	.780 712	1 219
.188	.622 937	2 022	.238	.712 532	1 570	.288	.781 931	1 213
.189	.624 959	2 012	.239	.714 102	1 562	.289	.783 144	1 206
.190	.626 971	2 002	.240	.715 664	1 555	.290	.784 350	1 201
.191	.628 973	1 992	.241	.717 219	1 546	.291	.785 551	1 193
.192	.630 965	1 982	.242	.718 765	1 538	.292	.786 744	1 188
.193	.632 947	1 972	.243	.720 303	1 531	.293	.787 932	1 182
.194	.634 919	1 962	.244	.721 834	1 522	.294	.789 114	1 175
.195	.636 881	1 952	.245	.723 356	1 515	.295	.790 289	1 169
.196	.638 833	1 943	.246	.724 871	1 507	.296	.791 458	1 163
.197	.640 776	1 933	.247	.726 378	1 500	.297	.792 621	1 157
.198	.642 709	1 923	.248	.727 878	1 492	.298	.793 778	1 151
.199	.644 632	1 913	.249	.729 370	1 484	.299	.794 929	1 145
.200	.646 545	1 904	.250	.730 854	1 476	.300	.796 074	1 139

Table V - Continued

Values of $_5q_x$ associated with $_5m_x$ by the equation:

$$_5q_x = 1 - e^{-5\,_5m_x - .008(5)^3\,_5m_x^2}$$

$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ	$_5m_x$	$_5q_x$	Δ
		.00			.000			.000
.300	.796 074	1 139	.350	.846 261	874	.400	.884 675	667
.301	.797 213	1 133	.351	.847 135	869	.401	.885 342	663
.302	.798 346	1 128	.352	.848 004	865	.402	.886 005	660
.303	.799 474	1 121	.353	.848 869	860	.403	.886 665	656
.304	.800 595	1 115	.354	.849 729	856	.404	.887 321	653
.305	.801 710	1 110	.355	.850 585	850	.405	.887 974	649
.306	.802 820	1 103	.356	.851 435	847	.406	.888 623	646
.307	.803 923	1 098	.357	.852 282	842	.407	.889 269	642
.308	.805 021	1 092	.358	.853 124	837	.408	.889 911	638
.309	.806 113	1 087	.359	.853 961	833	.409	.890 549	635
.310	.807 200	1 080	.360	.854 794	828	.410	.891 184	632
.311	.808 280	1 075	.361	.855 622	824	.411	.891 816	628
.312	.809 355	1 070	.362	.856 446	819	.412	.892 444	625
.313	.810 425	1 063	.363	.857 265	816	.413	.893 069	621
.314	.811 488	1 059	.364	.858 081	810	.414	.893 690	618
.315	.812 547	1 052	.365	.858 891	807	.415	.894 308	614
.316	.813 599	1 047	.366	.859 698	802	.416	.894 922	612
.317	.814 646	1 042	.367	.860 500	798	.417	.895 534	607
.318	.815 688	1 036	.368	.861 298	793	.418	.896 141	605
.319	.816 724	1 030	.369	.862 091	789	.419	.896 746	601
.320	.817 754	1 026	.370	.862 880	785	.420	.897 347	598
.321	.818 780	1 019	.371	.863 665	781	.421	.897 945	594
.322	.819 799	1 015	.372	.864 446	776	.422	.898 539	592
.323	.820 814	1 009	.373	.865 222	773	.423	.899 131	588
.324	.821 823	1 003	.374	.865 995	768	.424	.899 719	585
.325	.822 826	0 999	.375	.866 763	764	.425	.900 304	581
.326	.823 825	0 993	.376	.867 527	760	.426	.900 885	579
.327	.824 818	0 988	.377	.868 287	756	.427	.901 464	575
.328	.825 806	0 982	.378	.869 043	751	.428	.902 039	572
.329	.826 788	0 978	.379	.869 794	748	.429	.902 611	569
.330	.827 766	0 972	.380	.870 542	744	.430	.903 180	566
.331	.828 738	0 967	.381	.871 286	739	.431	.903 746	562
.332	.829 705	0 962	.382	.872 025	736	.432	.904 308	560
.333	.830 667	0 957	.383	.872 761	732	.433	.904 868	556
.334	.831 624	0 952	.384	.873 493	728	.434	.905 424	554
.335	.832 576	0 947	.385	.874 221	723	.435	.905 978	550
.336	.833 523	0 941	.386	.874 944	720	.436	.906 528	548
.337	.834 464	0 937	.387	.875 664	716	.437	.907 076	544
.338	.835 401	0 932	.388	.876 380	712	.438	.907 620	541
.339	.836 333	0 927	.389	.877 092	708	.439	.908 161	539
.340	.837 260	0 922	.390	.877 800	705	.440	.908 700	535
.341	.838 182	0 917	.391	.878 505	700	.441	.909 235	532
.342	.839 099	0 912	.392	.879 205	697	.442	.909 767	530
.343	.840 011	0 907	.393	.879 902	693	.443	.910 297	526
.344	.840 918	0 903	.394	.880 595	689	.444	.910 823	524
.345	.841 821	0 897	.395	.881 284	686	.445	.911 347	521
.346	.842 718	0 893	.396	.881 970	682	.446	.911 868	518
.347	.843 611	0 888	.397	.882 652	678	.447	.912 386	514
.348	.844 499	0 884	.398	.883 330	674	.448	.912 900	513
.349	.845 383	0 878	.399	.884 004	671	.449	.913 413	509
.350	.846 261	0 874	.400	.884 675	667	.450	.913 922	

Tables XIII and XIV demonstrate the computation of the different life table functions for Fiji Islands males and females respectively (1950-1952). The transformation of m_x into q_x have been effected with the help of the Reed and Merrell conversion tables.

Instructions for Completing Tables XIII and XIV: Computation of Abridged Life Tables.

In Table XIII, for example, m_o (i.e., age-sex specific death rate for the first year of age) is given to be 0.0654. [1] From the Reed-Merrell Table q_o, it is found that:

if $m_o = 0.065$, $q_o = 0.057930$ and

if $m_o = 0.066$, $q_o = 0.058761$

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[1] From Worksheet 16.

TABLE XIII

COMPUTATION OF ABRIDGED LIFE TABLE FOR MALES IN FIJI ISLANDS

1950 - 1952

AGE GROUP X THROUGH X + n (1)	AGE-SPECIFIC MORTALITY RATE (WORKSHEET 11) m_x (2)	LIFE TABLE MORTALITY RATE (MERRELL-REED TABLES) q_x (3)	NO. SURVIVING TO BEGINNING OF AGE INTERVAL FROM 100,000 LIVE BIRTHS. (COL. 4 LINE ABOVE MINUS COL. 5 LINE ABOVE) l_x (4)	NO. DYING IN INTERVAL X TO X + n (COL 3 X COL 4) d_x (5)	LIFE TABLE POPULATION IN INTERVAL X THROUGH X + n (COL. 5 ÷ COL. 2) L_x (6)	LIFE TABLE 5-YEAR SURVIVAL RATIOS (COL. 6 ONE ROW BELOW ÷ COL. 6 THIS ROW) $_5P_x$ (7)	LIFE TABLE 10-YEAR SURVIVAL RATIOS (COL. 6 2 ROWS BELOW ÷ COL. 6 THIS ROW) $_{10}P_x$ (8)	LIFE TABLE POPULATION AGED X AND OVER (Σ COL. 6 FOR THIS ROW AND ALL BELOW) T_x (9)	AVERAGE YEARS OF LIFE REMAINING TO SURVIVORS TO AGE X (COL. 9 ÷ COL. 4) e^o_x (10)
						P_b = 0.9290	P_{b_1} = 0.9290 P_{b_2} = 0.9044		
Less than 1	0.0654	0.0583	100,000	5,830	95,628 1/ --	0.9736	0.9642	5,878,994	58.80
1 through 4	0.0105	0.0395	94,170	3,720	368,868 2/ -			5,783,366	61.41
5 " 9	0.0018	0.0090	90,450	814	452,222	0.9903	0.9757	5,414,498	59.86
10 " 14	0.0014	0.0070	89,636	627	447,857	0.9852	0.9748	4,962,276	55.36
15 " 19	0.0024	0.0119	89,009	1,059	441,250	0.9894	0.9765	4,514,419	50.72
20 " 24	0.0029	0.0144	87,950	1,266	436,552	0.9870	0.9670	4,073,169	46.31
25 " 29	0.0034	0.0169	86,684	1,465	430,882	0.9797	0.9579	3,636,617	41.95
30 " 34	0.0042	0.0208	85,219	1,773	422,143	0.9777	0.9459	3,205,735	37.62
35 " 39	0.0055	0.0272	83,446	2,270	412,727	0.9675	0.9252	2,783,592	33.36
40 " 44	0.0074	0.0364	81,176	2,955	399,324	0.9563	0.8929	2,370,865	29.21
45 " 49	0.0102	0.0498	78,221	3,895	381,863	0.9337	0.8487	1,971,541	25.20
50 " 54	0.0182	0.0873	74,326	6,489	356,538	0.9090	0.7856	1,589,678	21.39
55 " 59	0.0198	0.0946	67,837	6,417	324,091	0.8643	0.7040	1,233,140	18.18
60 " 64	0.0405	0.1847	61,420	11,344	280,099	0.8146	0.6084	909,049	14.80
65 " 69	0.0408	0.1859	50,076	9,309	228,162	0.7469		628,950	12.56
70 " 74	0.0793	0.3315	40,767	13,514	170,416 — —	0.5748	0.3663	400,788	9.83
75 and over	0.1183	1.0000	27,253	27,253	230,372 —			230,372	8.45

1/ $L_0 = 25{,}000 + 0.75\ l_1$

2/ $L_{1\text{-}4} = 1.9\ l_1 + 2.1\ l_5$

TABLE XIV

COMPUTATION OF ABRIDGED LIFE TABLE FOR FEMALES IN FIJI ISLANDS

1950 - 1952

AGE GROUP	AGE-SPECIFIC MORTALITY RATE (WORKSHEET 11)	LIFE TABLE MORTALITY RATE (MERRELL-REED TABLES)	NO. SURVIVING TO BEGINNING OF AGE INTERVAL FROM 100,000 LIVE BIRTHS. (COL. 4 LINE ABOVE MINUS COL. 5 LINE ABOVE)	NO. DYING IN INTERVAL X TO X + n (COL. 3 X COL. 4)	LIFE TABLE POPULATION IN INTERVAL X THROUGH X + n (COL. 5÷ COL. 2)	LIFE TABLE 5-YEAR SURVIVAL RATIOS (COL. 6 ONE ROW BELOW ÷ COL. 6 THIS ROW)	LIFE TABLE 10-YEAR SURVIVAL RATIOS (COL. 6 2 ROWS BELOW ÷ COL. 6 THIS ROW)	LIFE TABLE POPULATION AGED X AND OVER (Σ COL. 6 FOR THIS ROW AND ALL BELOW)	AVERAGE YEARS OF LIFE REMAINING TO SURVIVORS TO AGE X (COL. 9 ÷ COL. 4)
X THROUGH X + n	m_x	q_x	l_x	d_x	L_x	$_5P_x$	$_{10}P_x$	T_x	e^o_x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
						$P_b = 0.9400$	$P_{b1} = 0.9400$ $P_{b2} = 0.9165$		
Less than 1	0.0523	0.0472	100.000	4,720	96,460 1/			5,762,867	57.63
1 through 4	0.0101	0.0380	95,280	3,621	373,516 2/	0.9750	0.9663	5,666,407	59.47
5 " 9	0.0017	0.0085	91,659	779	458,235	0.9910	0.9774	5,292,891	57.75
10 " 14	0.0017	0.0085	90,880	772	454,118	0.9863	0.9633	4,834,656	53.20
15 " 19	0.0033	0.0164	90,108	1,478	447,879	0.9767	0.9533	4,380,538	48.61
20 " 24	0.0047	0.0232	88,630	2,056	437,447	0.9760	0.9460	3,932,659	44.37
25 " 29	0.0059	0.0291	86,574	2,519	426,949	0.9693	0.9389	3,495,212	40.37
30 " 34	0.0065	0.0320	84,055	2,690	413,846	0.9686	0.9312	3,068,263	36.50
35 " 39	0.0069	0.0340	81,365	2,766	400,870	0.9613	0.9206	2,654,417	32.62
40 " 44	0.0082	0.0402	78,599	3,160	385,366	0.9576	0.9056	2,253,547	28.67
45 " 49	0.0093	0.0455	75,439	3,432	369,032	0.9457	0.8767	1,868,181	24.76
50 " 54	0.0137	0.0664	72,007	4,781	348,978	0.9270	0.8040	1,499,149	20.82
55 " 59	0.0165	0.0794	67,226	5,338	323,515	0.8673	0.6929	1,150,171	17.11
60 " 64	0.0429	0.1945	61,888	12,037	280,583	0.7990	0.5510	826,656	13.36
65 " 69	0.0467	0.2100	49,851	10,469	224,176	0.6897		546,073	10.95
70 " 74	0.1080	0.4240	39,382	16,698	154,611	0.5197	0.3063	321,897	8.17
75 and over	0.1356	1.0000	22,684	22,684	167,286			167,286	7.37

1/ $L_o = 25{,}000 + 0.75\, l_1$

2/ $L_{1\text{-}4} = 1.9\, l_1 + 2.1\, l_5$

Since 0.0654 is 4/10 of the interval between 0.065 and 0.066, it may be assumed that the corresponding q_o is also 4/10 of the interval between 0.057930 and 0.058761. The total distance between the two latter values is 0.000831 (extreme right column of Reed-Merrell table). Hence, the relevant $q_o = 0.057930 + 4/10\ (0.000831)$. That is, $q_o = 0.057930 + 0.000332$ and,

$$q_o = 0.0583$$ 1/

Transformation of m_1 (i.e., age-sex specific death rate for the group "1 through 4") may be effected in the same manner but with the use of the $_4q_1$ table. For example, in Table XIII, m_1 is found to be 0.0105 and and in the $_4q_1$ Reed-Merrell Table, it is found that:

if $m_1 = 0.010,\ _4q_1 = 0.037665$ and

if $m_1 = 0.011,\ _4q_1 = 0.041265$

Hence, by interpolation, it is found that for $m_1 = 0.105,\ _4q_1 = 0.0395$.

The transformation of the remaining five-year age-sex specific death rates into life table death rates may be done through the use of the $_5q_x$ table. In the example of Table XIII, for instance, the age-sex specific death rate for the age group "15 through 19" (i.e., m_{15}) is given at 0.0024. From the above mentioned Reed-Merrell table, it is found:

if $_5m_x = 0.002,\ _5q_x = 0.009954$ and

if $_5m_x = 0.003,\ _5q_x = 0.014897$

Hence, by interpolation:

when $_5m_{15} = 0.0024, \qquad _5q_{15} = 0.0119$

1/ Rounded to the fourth decimal place.

Finally, the q_x corresponding to the last age group--in this case, "75 and over"--is set at 1.0000 because it is an open end age group and all people will eventually die.

c) The l_x and d_x Functions: The l_x function assumes a hypothetical situation where 100,000 children are born alive every year. It then shows the probable number of these children surviving to the beginning of each age interval. The d_x function, on the other hand, indicates the probable number of deaths that each initial 100,000 births will experience while moving from one age group to another.

Hence, the first row of column (4) in Table XIII and Table XIV shows 100,000 live births. This is the value of l_o. The probability of death for this cohort, q_o, was found to be 0.0583 [column (3)]. Therefore, the probable number of deaths occurring among this cohort until reaching the beginning of the next age group--i.e., one year of age--is 100,000 x 0.0583 = 5830 which appears in the d_x column [column (5)] in the first row. It follows, therefore, that only 100,000 - 5830 = 94,170 survive to become one year of age [row 2, column (4)]. Of these, 0.0395 [column (3), row (2)] are likely to die before reaching the age of five years. The value in the second row of column (5)--i.e., the probable number of deaths of the given cohort--is thus:

$$94,170 \times 0.0395 = 3,720$$

and so on. It is seen therefore, that, beginning with a hypothetical 100,000 live births, we are able, with the help of the q_x values, to determine the most probable number of survivors (l_x) to the beginning of each age bracket, and the most probable number of deaths (d_x) that may occur among the initial 100,000 live births while moving through each age bracket.

d) The L_x Function: This function indicates the hypothetical age distribution of a population subjected to the conditions set by the previous functions--namely, the given mortality levels and the 100,000 live births each year.

The transformation of the l_x values into L_x values may be effected through short-cut methods devised in two different publications 1/ and combined in the present handbook. For x_{5-9} through x_{75+} --i.e., for all age groups except "less than 1" and "1 through 4"-- the L_x function may be directly obtained by dividing, for each row, the d_x by the m_x values. Thus for the age group "5 through 9" in Table XIII, the L_x value is 814/ 0.0018 = 452, 222. For the first two age groups, on the other hand, the following formulae may be used:

(1): $L_o = 25,000 + 0.75\ l_1$ and

(2): $L_{1\text{-}4} = 1.9\ l_1 + 2.1\ l_5$

Thus in Table XIII, it is found that, for the first year of age, $L_o = 25,000 + 0.75\ (94,170) = 95,628$; that is an L_x value of 95,628. For the age group "1 through 4", $L_{1\text{-}4} = 1.9\ (94,170) + 2.1\ (90,450) = 368,868$; hence, an L_x value of 368,868 persons.

e) The P_x Function: Columns (7) and (8) indicate the survival ratios of the hypothetical population given by the L_x function. The difference between them is that column (7), ($_5P_x$), indicates the five-year survival ratios--i.e., the proportion of a given cohort surviving to the next higher age group--while column (8), ($_{10}P_x$), gives the ten-year survival ratios, namely the proportion of a given cohort surviving to attain the age group ten years older. Computation in both cases is simple. For the five-year survival ratios, one may proceed as follows: If, as in column (6), out of 452, 222 persons "5 through 9", 447, 857 survive to the next higher age group "10 through 14"; then the five-year survival ratio of the former age

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1/ For the age groups "Less than 1" and "1 through 4", the method is described in United Nations, Methods of Population Projection by Age and Sex, op. cit., p. 23. For the other age groups, the method was devised in T.N.E. Greville, "Short Methods of Construction Abridged Life Tables", The Record of the American Institute of Actuaries, Vol. XXXll, June 1943. This latter article is also reprinted in A.J. Jaffe (ed), Handbook of Statistical Methods for Demographers (Op. cit.), pp. 28-34.

group is $\frac{447,857}{452,222} = 0.9903$; that is, 99.03 percent of the "5 through 9" cohort survive, under the given mortality conditions, to become "10 through 14". The other ${}_5P_x$ values may be similarly obtained.

The ${}_{10}P_x$ values, on the other hand, may be arrived at as follows: Out of 452, 222 persons "5 through 9", only 441, 250 survive for ten years to become "15 through 19". The ten-year survival ratio for the "5 through 9" age group is, hence $\frac{441,250}{452,222} = 0.9757$, or 97.57 percent.

The ${}_5P_b$ value shown on top of column (7) indicates the survival ratio of the 500, 000 live births that will reach the ages of "Less than 5". It is evident that out of these 500, 000 births (over the five-year period) only 464, 496 will live to be "Less than 5" [row 1 + row 2, column (6)] and their survival ratio is, hence, $\frac{464,496}{500,000} = 0.9290$

P_{b1} and P_{b2} of column (8) are obtained through a similar procedure. P_{b1} indicates the survival ratio of 500, 000 live births (over the five-year period) attaining the ages "Less than 5" and has, therefore, a value equal to $\frac{464,496}{500,000} = 0.9290$. P_{b2}, on the other hand, indicates the survival ratio of 500, 000 live births (over the five-year period) attaining the ages of "5 through 9". P_{b2} is, therefore, equal to $\frac{452,222}{500,000} = 0.9044$.

f) The T_x Function: This function, as well as the $e^o x$ function, succeed the P_x function in the sequence of computations of the life table. The T_x function indicates the total number of years yet to be lived by each cohort under the given mortality conditions. Its values may be obtained by adding the values of L_x from the bottom up through the appropriate cohort row. (See Column 9).

g) The e^o_x Function: This function is usually referred to as "life expectancy". Its values may be obtained by dividing, for each row, T_x by l_x. Thus the "life expectancy" at birth is given at 58.80 years (row 1, column 10), while "life expectancy" at the age of "20", for example, is set at 46.31 years (row 6, column 10).

Short-Cut Method for Estimating Life Table Survival Ratios

The estimator may, in some cases, be interested only in estimating $_5P_x$ rather than building a more or less comprehensive abridged life table. In such cases only two simple additional steps are required after the m_x and q_x functions have been obtained (see page 146).

WORKSHEET 18

SHORT-CUT METHOD OF COMPUTING SURVIVAL RATIOS FOR FIJI ISLANDS MALES 1950-1952

Age Group	Age-Sex Specific Mortality Rate m_x	Life Table Mortality Rate q_x	1-q_x p_x	Five-year Survival Ratios P_x
(1)	(2)	(3)	(4)	(5)
Less than 1	0.0654	0.0583	0.9417	0.9693
1 through 4	0.0105	0.0395	0.9605	
5 " 9	0.0018	0.0090	0.9910	0.9920
10 " 14	0.0014	0.0070	0.9930	0.9906
15 " 19	0.0024	0.0119	0.9881	0.9869
20 " 24	0.0029	0.0144	0.9856	0.9844
25 " 29	0.0034	0.0169	0.9831	0.9812
30 " 34	0.0042	0.0208	0.9792	0.9760
35 " 39	0.0055	0.0272	0.9728	0.9682
40 " 44	0.0074	0.0364	0.9636	0.9569
45 " 49	0.0102	0.0498	0.9502	0.9315
50 " 54	0.0182	0.0873	0.9127	0.9091
55 " 59	0.0198	0.0946	0.9054	0.8604
60 " 64	0.0405	0.1847	0.8153	0.8147
65 " 69	0.0408	0.1859	0.8141	0.7413
70 " 74	0.0793	0.3315	0.6685)	0.5746
75 and over	0.1183	1.0000	) -	

Instructions for Completing Worksheet 18: Short-cut Method of Computing Survival Ratios.

Columns (1), (2) and (3): These are exact replicas of columns (1), (2) and (3) of Table XIII.

Column (4): The values in this column may be obtained by substracting, for each row, the value of q_x from 1.0000.

Column (5): All values of P_x except P_b, P_{0-4} and P_{70+}, may be obtained by averaging each two successive values of p_x of column (4). For instance, P_{5-9} is obtained by adding $p_{5-9} = 0.9910$ to $p_{10-14} = 0.9930$ and dividing the result by 2 thus obtaining $P_{5-9} = 0.9920$.

P_b, P_{0-4} and P_{70+} may be obtained through direct application of the following formulae:

$$(1) \quad P_b = 0.05 + p_o\,(0.53 + 0.42\,p_{1-4})$$

$$(2) \quad P_{0-4} = \frac{(p_o \times p_{1-4})\,(1+p_{5-9})}{2P_b}$$

$$(3) \quad P_{70+} = 0.91\,P_{65-69} - 0.1$$

In case the open end age group is "85 and over" the $5P_x$ value will have to refer to the age group "80 and over". The relevant formula becomes:

$$(4) \quad P_{80+} = 0.8\,P_{65-69} - 0.1$$

Use of Model U. N. Life Tables

In some underdeveloped countries, death statistics are either too deficient or completely lacking so that the estimates of age-sex survival ratios, following the methods just expounded, may prove to be too unreliable for most purposes. This forces reliance upon data from another population whose mortality conditions are believed to be generally similar to those of the population under study. If such a population is found, its entire mortality schedule might better be substituted than model life tables developed for general application. However, where similar mortality conditions cannot be found, survival ratios may be computed with the use of a minimum amount of basic data supplemented by Model Life Tables published by the United Nations. [1] If the death rate of only one age-sex group, or the infant mortality rate p_b, [2] is known to be reliable, all the basic functions of a life table may be approximated with the help of the model tables. As an illustration, suppose that the only information we have on mortality conditions in the Fiji Islands is that the death rate of the male age group "25 through 29" in the year 1951 is 0.0034 (Table XIII), $1000\ m_{25}$ being equal to 3.4. From Table U.N.I., it is found that this age-sex specific mortality rate corresponds to a mortality "level" of between 80 and 85. The exact level may be determined by straight line interpolation thusly:

If $L = 80$, $1000\ m_{25} = 3.70$ (Table U.N.I., Column 18, row 7)

If $L = 85$, $1000\ m_{25} = 3.14$ (Table U.N.I., Column 19, row 7)

The difference between the first and the second $1000\ m_{25}$ is $3.70 - 3.14 = 0.56$ and the given $1000\ m_{25}$ is $\frac{3.70 - 3.40}{0.56} = \frac{0.30}{0.56}$ or $\frac{15}{28}$ of the distance between the two values starting from 3.70. The corresponding level is, therefore, assumed to be also $\frac{15}{28}$ the distance between 80 and 85 starting with 80, that is $L = 80 + \frac{15}{28}(85-80) = 83$ (rounded). Once this level is determined, the rest of the values of 1000 m_x may be obtained from the same table by interpolating between the values of 1000 m_x at level 80 and at level 85. The values of 1000 q_x, l_x, L_x, and P_x, may then be obtained in a similar manner from tables U.N. II, U.N. III, U.N. IV, and U.N. V. respectively. 1/

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[1] In Methods of Population Projections by Sex and Age, ST/SOA/ Ser. A, No. 25, and reproduced below.

[2] Or if similar information is available for a population believed to have similar mortality conditions as the population under study.

Two things should be emphasized here:

First, that the mortality data required to construct a life table with the help of the U. N. model may describe mortality of any age group including infant deaths and life expectancy at birth.

Second, that the results of this method are not usually sufficiently accurate to make them generally useful for estimating the growth components of population change between two censuses (Part II, Chapter II) but are generally well suited to carry on component population projections (Part II, Chapter III).

Census Survival Ratios 2/

We define a census survival ratio as "a fraction in which the numerator is the number of persons in an age-sex group of a closed population at a given census and the denominator is the number [x] years younger at the previous census" taken [x] years before. 3/ If there is reason to believe that migration to and from a given area or country is absent or negligible, or if migration is accurately known and its effect on the growth of population between two censuses can be eliminated, an estimate of the ratio of survivors from a given cohort at the first census may be obtained by dividing the size of this cohort at the second census (which is now [x] years older)by its size at the first census. For example, the census survival ratio for native white males (in the United States) aged 10-14 in 1930 and 20-24 in 1940 is:

$$\frac{\text{Native white males aged 20-24, U. S., 1940}}{\text{Native white males aged 10-14, U. S., 1930}}$$

1/ The values of 1000 m_x and 1000 q_x may then be divided by 1000 to make them comparable to those in Tables XIII and XIV.

2/ The following is a brief discussion of an alternative set of survival ratios, namely, those secured from census data. For a more detailed discussion of same see E. S. Lee, et. al., Op. cit. pp. 15ff.

3/ Ibid. p. 15.

In terms of actual data, this is:

$$\frac{5,014,725}{5,325,152} = 0.9417\text{''}$$ [1]

If the period between the two censuses is five years in length, the survival ratio of the births of those five years (corresponding to p_b in Life Table) must be obtained in a slightly different manner. One obvious method is to divide the size of the age group "less than 5" at the second census by the actual number of live births during the five preceding years. If the period is ten years, the size of the age group "Less than 5" in the second census may be divided by the actual number of live births during the five years preceding this census to obtain the survival ratio of that group (P_{b1}); and the age group "5 through 9" in the second census may be divided by the number of live births over the first five years of the ten inter-censal years to obtain the survival ratio of that group (P_{b2}). Thus, in our example:

$$P_{b1} = \frac{\text{Native white males "Less than 5", 1940}}{\text{Live births to native white males, 1935-1940}}$$

and

$$P_{b2} = \frac{\text{Native white males "5 through 9", 1940}}{\text{Live births to native white males, 1930-1935}}$$

Note first that census survival ratios do not attempt to reflect mortality accurately. Although they are exclusive of migration, they do reflect in addition to mortality of the different cohorts, the deficiencies in census statistics. In other words, the loss in the size of a cohort as reported in two censuses may not be due exclusively to mortality but also to miscounts in one or both censuses. For the younger ages where underenumeration is usually significant, the comparison between the size of the cohort at the first census and, (x) years later, at the next census, may show actual gain rather than loss. Therefore, the census survival ratio may in some instances, exceed 1.0000.

1/ Ibid.

Furthermore, it is obvious that census survival ratios, because they reflect (or assume) a situation where no migration took place, may not be used to calculate migration for the country or area for which the two censuses belong. For example, if one assumes in and out migration to India was negligible between two censuses and proceeds to compute census survival ratios for the period, one can't, at a later time, use these same ratios between the two given censuses to estimate migration. Obviously, such a use will show no migration, but only because a no-migration situation was assumed initially.

The main use of census survival ratios is for sub-national regions and for population projections. If it may be assumed that a sub-national region has similar mortality conditions as the national entity for which the census survival ratios have been computed, these ratios may be used to estimate migration and natural increase in the region following the method described in the last section of Part II, Chapter II. Furthermore, national and sub-national regions may use these census survival ratios, or a projection thereof, for projecting population into the future (Part II, Chapter III).

Projecting Life Table Survival Ratios

Until now the discussion has covered the computation of life table survival ratios for past periods. These ratios may be used in estimating natural increase and migration between two censuses (Part II, Chapter II) but they are generally not used for making component projections into the future. 1/ In developing countries, the level of mortality for all age-sex groups is usually a declining function of time. If age-sex specific survival ratios are available for more than one point in time, it will generally be found that they are rising. In such instances, these ratios may be extrapolated to obtain ratios relevant to future periods and hence for projection purposes.

Another method of projecting survival ratios (most suitable for situations where past survival ratios are available for only one point in time) is embodied in the United Nations Model Life Tables. It should be noted that the stated

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1/ The "future" is still taken as the period past the last census.

levels in the United Nations Tables are of five unit spans starting with an arbitary level of zero (0, 5, 10...115). Each unit may be considered as one year in length so that each stated level is five years distant from the preceding level as well as from the succeeding level. If, as in the example of section C of this chapter, it is determined that in the year [x], a given country's mortality conditions correspond to level 83, at the year [x + n], this mortality level may be assumed to coincide with mortality level [83 + n], and a life table may thus be constructed. Furthermore, if the initial life table is constructed from actual mortality rates as in section C of this chapter and age-sex survival ratios are found to correspond to more than one mortality level, they may nevertheless be projected with the help of the United Nations Model Life Tables. In this case, each age-sex group mortality level has to be first determined and these levels may then be projected individually by adding the number of years (n) of the projection period to each individual level. The step by step technique of so doing is illustrated in Worksheets 19 and 20 with reference to the Fiji Islands' male population. In Worksheet 19, it is assumed that the $_5P_x$ values obtained in Table XIII are reasonably accurate and no attempt is made to adjust them. In Worksheet 20 this assumption is dropped and adjustment of these ratios is attempted in column (4) with the help of the Model Life Tables. 1/

1/ In the case at hand there seems to be need for such an adjustment. Note, for instance, that the survival ratio for the age group "10 through 14" is lower than that for the age groups "15 through 19" and "20 through 24". The fact that the survival ratios increase at the beginning (first three rows) is, however, consistent with observation in view of the fact that mortality is usually relatively high and falling over the first few years of life.

WORKSHEET 19

PROJECTING SURVIVAL RATIOS, WITHOUT ADJUSTMENT, FIJI ISLANDS, MALES, 1951 to 1956

Age Group	Life Table $5P_x$ values (1951) (Table XIII)	Mortality level Mx_1 (Table U.N.V.)	Mortality Level M_{x2} $(M_1 + 5)$	Life Table $5P_x$ values Projected to 1956
(1)	(2)	(3)	(4)	(5)
P_b	0.9290	86	91	.9466
Less than 5	0.9736	82	87	.9786
5 through 9	0.9903	78	83	.9918
10 " 14	0.9852	66	71	.9870
15 " 19	0.9894	90	95	.9913
20 " 24	0.9870	89	94	.9893
25 " 29	0.9797	78	83	.9828
30 " 34	0.9777	78	83	.9810
35 " 39	0.9675	71	76	.9718
40 " 44	0.9563	69	74	.9609
45 " 49	0.9337	63	68	.9395
50 " 54	0.9090	63	68	.9162
55 " 59	0.8643	58	63	.8731
60 " 64	0.8146	60	65	.8244
65 " 69	0.7469	64	69	.7576
70 and over	0.5748	94	99	.5808

Instructions for Completing Worksheet 19: Projecting Survival Ratios Without Adjustment.

Column (1): List age groups in five-year age brackets ending with age bracket "70 and over".

Column (2): List the $5P_x$ values obtained in Table XIII.

Column (3): By comparing each age-sex survival ratio in column (2) with the corresponding survival ratios of Table U. N. V, determine the appropriate mortality level Mx_1. For instance, from column (2) we find that the $5P_x$ value for the age group "25 through 29" is given to be 0.9797. From Table U. N. V (row four), it is found that this survival ratio coincides with a mortality level below 80 where $5P_{25-29} = 0.9811$ but above 75 where $5P_{25-29} = 0.9781$. In fact, $5P_{25-29} = 0.9797$ falls about 3/5 of the way between the two latter values starting from the smaller one. [1] The corresponding mortality level is, therefore, equal to 78. Because the last age group is "70 and over" rather than "80 and over" as in the U. N. Tables, projecting $5P_{70+}$ must be handled in a slightly different manner. In column (2) of this Worksheet, it is found that $5P_{70+} = 0.5748$. From Table U. N. IV we may obtain $5P_{70+}$ for any level by dividing the sum of the last three rows by the sum of the last four rows. Such an operation shows that for mortality level 90, $5P_{70+} = 0.5697 (< 0.5748)$ and that for mortality level 95, $5P_{70+} = 0.5759 (> 0.5748)$. Again, by interpolation, we find that the mortality level coinciding with $5P_{70+} = 0.5748$ is equal to 94.

Column (4): Because in this instance, survival ratios are being projected to 1956, i.e., five years past the initial life table date, each level in column (3) is augmented by five to obtain the mortality levels of 1956 for this column.

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[1] This is found by interpolation: The distance between $5P_{25} = 0.9811$ and $5P_{25} = 0.9781$ is $0.9811 - 0.9781 = 0.0030$. The distance between the latter $5P_{25}$ value and $5P_{25} = 0.9797$ is $0.9797 - 0.9781 = 0.0016$ or $\frac{16}{30}$ (i.e., little less than 3/5) of the former distance. In terms of levels the former distance is five units so that $5P_{25} = 0.9797$ coincides with a mortality level equal to $75 + 16/30\ (5) = 78$.

Column (5): From Table U. N. V, find the $_5P_x$ value corresponding to each age group mortality level of column (4). For example, the mortality level for the age group "30 through 34" in column (4) is given at 83. This is 3/5 of the distance between level 80 and level 85. For level 80 in Table U. N. V, $_5P_{30} = 0.9792$ and for level 85, $_5P_{30} = 0.9822$. Hence, $_5P_{30}$ for level 83 is equal to $0.9792 + 3/5\,(0.9822-0.9792) = 0.9810$. For the final age group "70 and over" recourse to Table U. N. IV must again be made to find the value $_5P_{70+}$ for level 99. From this table, it is found that $_5P_{70+}$ for level 95 is 0.5759 and level 100 is 0.5820. Because level 99 is 4/5 of the distance between 95 and 100, starting with 95, the corresponding $_5P_{70+}$ may be assumed to be also 4/5 of the distance between 0.5759 and 0.5820. Hence:

$$_5P_{70+} = 0.5759 + 4/5\,(0.5820 - 0.5759) = 0.5808.$$

WORKSHEET 20

PROJECTING SURVIVAL RATIOS, WITH ADJUSTMENT, FIJI ISLANDS, MALES, 1951 to 1956

Age Group	Life Table $_5P_x$ Values (Table XIII)	Mortality Level M_{x1} (Table U.N. V)	Mortality Level M_{xa} Adjusted	Mortality Level M_{x2} (M_{xa}+5)	Life Table $_5P_x$ Values Projected
(1)	(2)	(3)	(4)	(5)	(6)
P_b	0.9290	86	78	83	.9185
Less than 5	0.9736	82	78	83	.9742
5 through 9	0.9903	78	78	83	.9918
10 " 14	0.9852	66	78	83	.9908
15 " 19	0.9894	90	84	89	.9889
20 " 24	0.9870	89	84	89	.9868
25 " 29	0.9797	78	84	89	.9861
30 " 34	0.9777	78	84	89	.9844
35 " 39	0.9675	71	70	75	.9711
40 " 44	0.9563	69	70	75	.9617
45 " 49	0.9337	63	67	72	.9439
50 " 54	0.9090	63	67	72	.9213
55 " 59	0.8643	58	67	72	.8876
60 " 64	0.8146	60	67	72	.8372
65 " 69	0.7469	64	67	72	.7638
70 and over	0.5748	94	67	72	.5443

Instructions for Completing Worksheet 20: Projecting Survival Ratios with Adjustment.

Columns (1), (2) and (3): Same as in Worksheet 19.

Column (4): Going down column (3), it is found that the first four values show a remarkable difference from the next four values, and that these, in turn, differ substantially from the next two values, and that these latter values, finally, deviate appreciably from the remaining values. The average for each group has been obtained and assumed to apply to each individual item in the group as shown in this column.

Column (5): Add five units to the "adjusted" levels of column (4) of this worksheet.

Column (6): Obtained by same procedure as column (5) in Worksheet 19.

United Nations Tables

Number

I II III IV and V

From
Manual III
Methods For Population Projections
by Sex and Age
United Nations
ST/SOA/Series A
Population Studies, No. 25

TABLE I. AGE-SPECIFIC DEATH RATES

Mortality level (or time-

Sex and age (x) in years	*Level 0* (${}^{o}e_0 = 20$)	*Level 5* (${}^{o}e_0 = 22.5$)	*Level 10* (${}^{o}e_0 = 25$)	*Level 15* (${}^{o}e_0 = 27.5$)	*Level 20* (${}^{o}e_0 = 30$)	*Level 25* (${}^{o}e_0 = 32.5$)	*Level 30* (${}^{o}e_0 = 35$)	*Level 35* (${}^{o}e_0 = 37.5$)	*Level 40* (${}^{o}e_0 = 40$)	*Level 45* (${}^{o}e_0 = 42.5$)	*Level 50* (${}^{o}e_0 = 45$)	*Level 55* (${}^{o}e_0 = 47.5$)
MALES												
0	442.63	411.67	371.41	341.60	316.20	293.25	70.17	248.20	229.41	211.29	193.65	177.35
1–4	77.96	67.11	58.27	50.98	44.99	39.73	35.13	30.97	27.55	24.31	21.44	18.85
5–9	18.64	16.21	14.16	12.44	11.00	9.72	8.58	7.55	6.71	5.90	5.20	4.56
10–14	11.45	10.09	8.92	7.91	7.05	6.28	5.59	4.96	4.43	3.92	3.47	3.07
15–19	14.09	12.88	11.75	10.68	9.76	8.90	8.05	7.25	6.57	5.91	5.32	4.79
20–24	17.93	16.66	15.41	14.21	13.20	12.23	11.17	10.21	9.32	8.47	7.67	6.93
25–29	21.13	19.38	17.70	16.12	14.73	13.43	12.13	10.98	9.94	8.96	8.04	7.22
30–34	25.43	22.95	20.66	18.55	16.74	15.07	13.48	12.08	10.84	9.69	8.64	7.70
35–39	31.77	28.20	25.01	22.15	19.76	17.59	15.59	13.87	12.38	11.01	9.77	8.67
40–44	40.65	35.72	31.40	27.62	24.50	21.71	19.20	17.06	15.17	13.45	11.96	10.66
45–49	50.53	44.37	39.05	34.41	30.54	27.10	24.07	21.52	19.22	17.14	15.38	13.85
50–54	59.69	53.02	47.17	42.02	37.74	33.92	30.43	27.46	24.82	22.49	20.36	18.56
55–59	71.38	64.55	58.34	52.73	47.98	43.66	39.64	36.14	33.01	30.18	27.68	25.49
60–64	84.96	78.49	72.35	66.59	61.60	56.97	52.48	48.48	44.87	41.57	38.65	36.12
65–69	106.42	100.14	93.94	87.92	82.65	77.63	72.48	67.82	63.58	59.64	56.00	52.77
70–74	144.12	137.21	130.28	123.34	116.91	110.72	104.51	98.77	93.31	88.21	83.43	79.17
75–79	194.69	186.80	178.81	170.84	163.60	156.14	148.65	141.69	135.04	128.82	122.91	117.55
80–84	274.44	264.86	254.17	243.76	234.75	225.25	215.65	206.60	198.10	190.09	182.54	175.67
85+	511.36	460.12	418.80	387.42	363.99	344.56	327.70	313.93	302.48	292.58	284.10	276.97
FEMALES												
0	398.43	365.17	335.65	309.22	283.41	259.87	239.81	220.56	202.22	184.42	167.81	151.94
1–4	79.80	68.66	59.60	52.13	45.75	40.16	35.51	31.29	27.71	24.33	21.37	18.69
5–9	19.44	16.90	14.76	12.96	11.41	10.02	8.85	7.79	6.89	6.03	5.29	4.61
10–14	13.10	11.54	10.19	9.03	8.02	7.10	6.31	5.59	4.98	4.39	3.87	3.40
15–19	15.97	14.59	13.29	12.09	10.93	9.86	8.92	8.02	7.21	6.42	5.70	5.02
20–24	19.91	18.50	17.10	15.77	14.35	13.02	11.88	10.85	9.73	8.67	7.72	6.83
25–29	23.98	21.98	20.07	18.28	16.52	14.89	13.46	12.17	10.82	9.56	8.38	7.37
30–34	28.30	25.52	22.96	20.61	18.40	16.38	14.64	13.11	11.56	10.14	8.91	7.81
35–39	33.17	29.44	26.09	23.10	20.40	17.96	15.92	14.15	12.41	10.83	9.51	8.34
40–44	37.24	32.75	28.82	25.36	22.29	19.54	17.29	15.36	13.55	11.90	10.50	9.27
45–49	42.25	37.18	32.79	28.95	25.59	22.61	20.11	17.99	16.01	14.22	12.67	11.30
50–54	49.17	43.80	39.06	34.86	31.03	27.62	24.82	22.42	20.10	18.02	16.30	14.70
55–59	57.84	52.49	47.57	43.08	38.86	35.03	31.86	29.08	26.36	23.88	21.74	19.81
60–64	72.71	67.28	62.15	57.29	52.49	48.07	44.34	41.00	37.64	34.56	31.76	29.18
65–69	93.87	88.49	83.15	77.92	72.47	67.35	62.99	58.99	54.81	50.94	47.43	44.21
70–74	129.46	123.41	117.29	111.22	104.86	98.83	93.42	88.36	83.13	78.25	73.71	69.49
75–79	183.54	176.33	168.86	161.47	153.68	145.74	138.89	132.45	125.61	119.19	113.20	107.63
80–84	261.12	251.93	242.11	232.48	222.14	211.79	202.85	194.50	185.42	176.94	169.01	161.57
85+	456.14	417.09	386.93	362.32	341.38	323.96	310.57	299.33	288.89	279.84	272.19	265.53

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[a] Equivalent values of ${}^{o}e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

(1,000 m_x) OF MODEL LIFE TABLES

reference in years) [a]

Level 60 ($^oe_0$ = *50*)	*Level 65* ($^oe_0$ = *52.5*)	*Level 70* ($^oe_0$ = *55*)	*Level 75* ($^oe_0$ = *57.6*)	*Level 80* ($^oe_0$ = *60.4*)	*Level 85* ($^oe_0$ = *63.2*)	*Level 90* ($^oe_0$ = *65.8*)	*Level 95* ($^oe_0$ = *68.2*)	*Level 100* ($^oe_0$ = *70.2*)	*Level 105* ($^oe_0$ = *71.7*)	*Level 110* ($^oe_0$ = *73.0*)	*Level 115* ($^oe_0$ = *73.9*)	*Sex and age* (x) *in years*
												MALES
161.16	145.05	129.59	110.85	90.18	70.10	52.51	39.13	30.35	24.55	20.73	18.18	0
16.44	14.15	12.10	10.18	8.45	6.79	5.20	3.67	2.45	1.64	1.11	0.75	1–4
3.98	3.44	2.96	2.51	2.12	1.76	1.45	1.15	0.81	0.56	0.39	0.28	5–9
2.70	2.34	2.05	1.77	1.53	1.30	1.09	0.89	0.66	0.47	0.35	0.26	10–14
4.29	3.78	3.32	2.90	2.51	2.14	1.81	1.49	1.13	0.80	0.59	0.44	15–19
6.22	5.48	4.84	4.22	3.62	3.04	2.52	2.02	1.49	1.08	0.80	0.61	20–24
6.45	5.68	4.97	4.30	3.70	3.14	2.59	2.08	1.57	1.18	0.92	0.75	25–29
6.84	6.00	5.24	4.55	3.91	3.33	2.79	2.29	1.82	1.43	1.18	1.00	30–34
7.70	6.77	5.94	5.19	4.50	3.88	3.31	2.79	2.31	1.90	1.63	1.45	35–39
9.50	8.41	7.45	6.58	5.79	5.07	4.45	3.87	3.35	2.88	2.52	2.27	40–44
12.49	11.22	9.97	9.07	8.16	7.33	6.61	5.92	5.32	4.71	4.17	3.76	45–49
16.95	15.43	14.08	12.85	11.78	10.81	9.96	9.14	8.40	7.67	6.95	6.30	50–54
23.52	21.65	20.00	18.53	17.19	15.98	14.93	13.99	13.02	12.08	11.22	10.37	55–59
33.83	31.58	29.53	27.69	26.05	24.59	23.24	21.96	20.70	19.42	18.22	17.13	60–64
49.82	46.92	44.32	42.02	39.85	37.92	36.13	34.42	32.74	31.02	29.46	28.01	65–69
75.35	71.61	68.14	65.01	62.09	59.41	56.94	54.60	52.36	50.13	48.10	46.28	70–74
112.56	107.64	103.28	99.39	95.73	92.40	89.26	86.32	83.35	80.36	77.33	74.56	75–79
169.30	162.98	157.07	151.67	146.64	141.99	137.74	133.68	129.39	126.39	122.42	118.21	80–84
270.72	264.90	259.83	255.32	251.20	247.52	244.26	241.34	238.71	236.36	234.24	232.36	......85+
												FEMALES
136.41	121.72	107.62	91.92	74.54	57.57	42.65	31.27	23.79	18.84	15.57	13.39	0
16.19	13.87	11.65	9.80	7.92	6.14	4.55	3.09	2.06	1.38	0.93	0.63	1–4
3.99	3.41	2.90	2.42	1.97	1.56	1.18	0.85	0.60	0.42	0.29	0.21	5–9
2.96	2.55	2.17	1.82	1.49	1.18	0.91	0.67	0.50	0.36	0.26	0.20	10–14
4.38	3.79	3.27	2.77	2.30	1.86	1.45	1.10	0.84	0.60	0.44	0.33	15–19
5.97	5.19	4.46	3.76	3.13	2.54	1.99	1.49	1.11	0.80	0.60	0.47	20–24
6.45	5.58	4.80	4.06	3.38	2.73	2.19	1.70	1.29	0.97	0.76	0.62	25–29
6.80	5.89	5.09	4.34	3.64	3.00	2.47	1.97	1.56	1.23	1.00	0.85	30–34
7.28	6.31	5.48	4.70	4.00	3.37	2.84	2.35	1.95	1.60	1.38	1.22	35–39
8.15	7.15	6.29	5.49	4.78	4.14	3.58	3.06	2.65	2.31	2.04	1.87	40–44
10.06	8.95	8.00	7.11	6.33	5.62	5.02	4.45	4.00	3.62	3.28	3.02	45–49
13.22	11.91	10.78	9.74	8.80	7.94	7.20	6.51	5.98	5.51	5.15	4.83	50–54
18.05	16.48	15.05	13.71	12.53	11.45	10.45	9.52	8.87	8.33	7.92	7.62	55–59
26.79	24.70	22.82	21.04	19.41	17.89	16.58	15.33	14.47	13.78	13.10	12.59	60–64
41.19	38.47	35.91	33.46	31.26	29.21	27.41	25.71	24.47	23.39	22.33	21.29	65–69
65.38	61.59	58.10	54.80	51.80	48.96	46.46	44.08	42.31	40.53	38.68	36.89	70–74
102.30	97.39	92.63	88.03	83.82	79.85	76.31	72.96	70.52	68.24	65.40	62.47	75–79
154.48	147.96	141.94	136.17	130.92	125.92	121.48	117.21	113.98	110.99	107.61	104.01	80–84
259.56	254.36	249.76	245.57	241.69	238.20	235.22	232.54	230.61	228.97	227.41	225.98	......85+

TABLE II. LIFE-TABLE DEATH RATES

Mortality level (or time-

Sex and age (x) in years	*Level 0* ($\mathring{e}_0$ = *20*)	*Level 5* ($\mathring{e}_0$ = *22.5*)	*Level 10* ($\mathring{e}_0$ = *25*)	*Level 15* ($\mathring{e}_0$ = *27.5*)	*Level 20* ($\mathring{e}_0$ = *30*)	*Level 25* ($\mathring{e}_0$ = *32.5*)	*Level 30* ($\mathring{e}_0$ = *35*)	*Level 35* ($\mathring{e}_0$ = *37.5*)	*Level 40* ($\mathring{e}_0$ = *40*)	*Level 45* ($\mathring{e}_0$ = *42.5*)	*Level 50* ($\mathring{e}_0$ = *45*)	*Level 55* ($\mathring{e}_0$ = *47.5*)
MALES												
0	332.31	310.55	290.49	271.93	255.59	240.38	224.65	209.25	195.73	182.39	169.09	156.53
1–4	267.98	235.27	207.67	184.21	164.43	146.69	130.86	116.31	104.16	92.50	82.07	72.52
5–9	89.06	77.89	68.40	60.33	53.53	47.42	42.01	37.06	33.00	29.09	25.65	22.55
10–14	55.68	49.22	43.61	38.78	34.66	30.93	27.57	24.47	21.89	19.42	17.22	15.24
15–19	68.08	62.41	57.05	52.03	47.65	43.52	39.45	35.60	32.34	29.14	26.28	23.68
20–24	85.81	79.96	74.20	68.61	63.91	59.35	54.34	49.76	45.52	41.46	37.63	34.07
25–29	100.33	92.40	84.74	77.49	71.05	64.96	58.89	53.45	48.50	43.81	39.40	35.44
30–34	119.56	108.54	98.22	88.66	80.35	72.62	65.19	58.61	52.79	47.33	42.27	37.76
35–39	147.14	131.73	117.68	104.93	94.14	84.24	75.03	67.03	60.05	53.58	47.67	42.45
40–44	184.52	163.96	145.60	129.18	115.45	102.96	91.58	81.81	73.06	65.05	58.06	51.94
45–49	224.27	199.69	177.89	158.44	141.87	126.88	113.54	102.09	91.67	82.19	74.04	66.96
50–54	259.66	234.09	210.97	190.11	172.43	156.31	141.39	128.48	116.84	106.46	96.89	88.69
55–59	302.86	277.92	254.60	232.95	214.20	196.83	180.32	165.73	152.50	140.30	129.43	119.81
60–64	350.36	328.05	306.35	285.41	266.90	249.36	231.93	216.20	201.75	188.28	176.23	165.63
65–69	420.21	400.41	380.42	360.40	342.45	325.04	306.80	289.97	274.30	259.50	245.62	233.12
70–74	529.66	510.83	491.26	471.32	452.34	433.58	414.31	396.07	378.29	361.39	345.17	330.47
75–79	654.67	636.83	618.02	598.55	580.59	561.54	541.86	523.16	504.82	487.20	470.10	454.28
80–84	813.99	796.15	777.25	757.58	739.73	720.64	700.45	681.17	662.40	644.34	626.70	610.36
FEMALES												
0	306.76	286.66	268.15	251.01	233.73	217.48	203.25	189.25	175.59	162.01	149.05	136.40
1–4	273.39	240.03	211.87	187.93	166.95	148.16	132.18	117.46	104.75	92.58	81.81	71.93
5–9	92.69	81.06	71.16	62.79	55.45	48.87	43.29	38.19	33.85	29.70	26.08	22.81
10–14	63.42	56.06	49.69	44.18	39.29	34.87	31.08	27.58	24.58	21.70	19.16	16.85
15–19	76.77	70.38	64.32	58.68	53.21	48.11	43.60	39.32	35.42	31.61	28.10	24.79
20–24	94.84	88.38	82.02	75.84	69.27	63.03	57.70	52.80	47.48	42.44	37.88	33.58
25–29	113.13	104.19	95.57	87.38	79.35	71.80	65.10	59.03	52.67	46.68	41.02	36.18
30–34	132.14	119.96	108.56	97.99	87.96	78.66	70.62	63.45	56.19	49.43	43.57	38.31
35–39	153.14	137.11	122.47	109.21	97.05	85.94	76.53	68.32	60.19	52.70	46.43	40.86
40–44	170.34	151.35	134.41	119.24	105.55	93.15	82.87	73.98	65.52	57.77	51.15	45.28
45–49	191.05	170.10	151.54	134.96	120.27	106.99	95.75	86.09	76.96	68.68	61.40	54.93
50–54	218.97	197.41	177.92	160.33	144.01	129.18	116.86	106.14	95.70	86.19	78.28	70.92
55–59	252.65	232.02	212.56	194.48	177.08	161.05	147.54	135.54	123.64	112.68	103.07	94.38
60–64	307.59	288.00	268.95	250.56	232.02	214.56	199.56	185.95	172.01	159.03	147.09	135 97
65–69	380.19	362.28	344.19	326.08	306.78	288.25	272.07	257.04	241.02	225.91	212.00	199.06
70–74	488.91	471.53	453.48	435.07	415.44	396.24	378.63	361.87	344.11	327.22	311.21	296.02
75–79	629.00	611.86	593.79	575.08	555.05	534.15	515.43	497.53	477.96	459.17	441.16	424.03
80–84	789.92	772.63	754.27	735.19	714.27	692.37	672.99	654.29	633.46	613.38	594.02	575.44

[a] Equivalent values of $\mathring{e}_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

($1{,}000q_x$) OF MODEL LIFE TABLES

reference in years) [a]

Level 60 ($^{\circ}e_0 = 50$)	Level 65 ($^{\circ}e_0 = 52.5$)	Level 70 ($^{\circ}e_0 = 55$)	Level 75 ($^{\circ}e_0 = 57.6$)	Level 80 ($^{\circ}e_0 = 60.4$)	Level 85 ($^{\circ}e_0 = 63.2$)	Level 90 ($^{\circ}e_0 = 65.8$)	Level 95 ($^{\circ}e_0 = 68.2$)	Level 100 ($^{\circ}e_0 = 70.2$)	Level 105 ($^{\circ}e_0 = 71.7$)	Level 110 ($^{\circ}e_0 = 73.0$)	Level 115 ($^{\circ}e_0 = 73.9$)	Sex and age (x) in years
												MALES
143.78	130.82	118.11	102.34	84.47	66.60	50.52	38.01	29.67	24.11	20.41	17.94	0
63.56	54.98	47.22	39.88	33.21	26.78	20.56	14.55	9.75	6.55	4.42	3.00	1–4
19.72	17.03	14.68	12.48	10.55	8.75	7.20	5.73	4.06	2.80	1.96	1.40	5–9
13.40	11.66	10.18	8.81	7.61	6.48	5.44	4.49	3.31	2.36	1.73	1.31	10–14
21.22	18.74	16.49	14.39	12.45	10.64	8.99	7.42	5.64	4.01	2.93	2.21	15–19
30.63	27.02	23.90	20.89	17.94	15.10	12.52	10.07	7.44	5.37	3.99	3.07	20–24
31.74	28.00	24.54	21.25	18.35	15.59	12.87	10.34	7.83	5.90	4.61	3.75	25–29
33.61	29.56	25.87	22.48	19.36	16.49	13.86	11.39	9.05	7.14	5.86	5.01	30–34
37.75	33.28	29.28	25.60	22.26	19.22	16.43	13.85	11.50	9.47	8.12	7.22	35–39
46.41	41.17	36.57	32.36	28.52	25.04	22.02	19.16	16.61	14.30	12.50	11.30	40–44
60.58	54.57	48.66	44.36	40.00	36.00	32.50	29.15	26.23	23.30	20.66	18.61	45–49
81.28	74.28	67.99	62.25	57.21	52.65	48.57	44.67	41.11	37.63	34.15	31.01	50–54
111.05	102.67	95.26	88.53	82.41	76.85	71.96	67.56	63.06	58.62	54.58	50.54	55–59
155.94	146.35	137.49	129.48	122.30	115.83	109.82	104.10	98.40	92.61	87.14	82.15	60–64
221.49	209.98	199.52	190.11	181.21	173.20	165.69	158.46	151.32	143.92	137.21	130.87	65–69
317.01	303.71	291.13	279.61	268.74	258.62	249.22	240.20	231.51	222.76	214.70	207.39	70–74
439.18	424.08	410.44	398.05	386.23	375.29	364.86	354.98	344.87	334.57	323.99	314.24	75–79
594.78	578.98	563.92	549.84	536.52	523.97	512.30	500.96	490.21	480.19	468.67	456.23	80–84
												FEMALES
123.75	111.54	99.58	85.99	70.59	55.19	41.33	30.55	23.37	18.58	15.39	13.26	0
62.64	53.92	45.50	38.41	31.17	24.24	18.03	12.27	8.22	5.52	3.72	2.52	1–4
19.73	16.88	14.39	12.03	9.82	7.77	5.89	4.23	3.01	2.08	1.46	1.05	5–9
14.68	12.65	10.81	9.05	7.40	5.86	4.53	3.36	2.49	1.79	1.32	1.01	10–14
21.65	18.78	16.22	13.74	11.43	9.24	7.25	5.48	4.17	2.99	2.20	1.67	15–19
29.42	25.59	22.06	18.63	15.54	12.61	9.89	7.44	5.51	4.01	3.01	2.35	20–24
31.75	27.51	23.73	20.09	16.74	13.55	10.88	8.47	6.41	4.83	3.78	3.08	25–29
33.45	29.01	25.11	21.44	18.04	14.91	12.25	9.81	7.79	6.12	5.00	4.26	30–34
35.73	31.08	27.03	23.23	19.81	16.69	14.11	11.68	9.70	7.99	6.85	6.09	35–39
39.94	35.11	30.94	27.06	23.62	20.48	17.76	15.21	13.19	11.49	10.17	9.29	40–44
49.06	43.80	39.20	34.93	31.17	27.71	24.78	21.99	19.79	17.94	16.28	14.99	45–49
63.98	57.83	52.50	47.53	43.06	38.91	35.37	32.01	29.47	27.15	25.41	23.85	50–54
86.36	79.12	72.54	66.29	60.76	55.65	50.91	46.47	43.37	40.78	38.84	37.38	55–59
125.54	116.32	107.95	99.95	92.57	85.61	79.59	73.84	69.81	66.61	63.41	61.01	60–64
186.71	175.46	164.76	154.38	144.98	136.09	128.27	120.77	115.32	110.49	105.77	101.05	65–69
280.98	266.86	253.73	241.01	229.29	218.09	208.12	198.52	191.33	184.00	176.35	168.88	70–74
407.31	391.61	376.08	360.75	346.50	332.80	320.43	308.53	299.75	291.47	281.04	270.15	75–79
557.17	540.05	523.83	507.96	493.16	478.85	465.91	453.25	443.51	434.43	423.99	412.71	80–84

TABLE III. SURVIVORS TO EXACT AGES

Mortality level or (time-

Sex and age (x) in years	*Level 0* ($^{o}e_0$ = *20*)	*Level 5* ($^{o}e_0$ = *22.5*)	*Level 10* ($^{o}e_0$ = *25*)	*Level 15* ($^{o}e_0$ = *27.5*)	*Level 20* ($^{o}e_0$ = *30*)	*Level 25* ($^{o}e_0$ = *32.5*)	*Level 30* ($^{o}e_0$ = *35*)	*Level 35* ($^{o}e_0$ = *37.5*)	*Level 40* ($^{o}e_0$ = *40*)	*Level 45* ($^{o}e_0$ = *42.5*)	*Level 50* ($^{o}e_0$ = *45*)	*Level 55* ($^{o}e_0$ = *47.5*)
MALES												
0………	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1………	66,769	68,545	70,951	72,807	74,441	75,962	77,535	79,075	80,427	81,761	83,091	84,347
5………	48,876	52,418	56,217	59,395	62,201	64,819	67,389	69,878	72,050	74,198	76,272	78,230
10………	44,523	48,335	52,372	55,812	58,871	61,745	64,558	67,288	69,672	72,040	74,316	76,466
15………	42,044	45,956	50,088	53,648	56,831	59,835	62,778	65,641	68,147	70,641	73,036	75,301
20………	39,182	43,088	47,230	50,857	54,123	57,231	60,301	63,304	65,943	68,583	71,117	73,518
25………	35,820	39,643	43,726	47,368	50,664	53,834	57,024	60,154	62,941	65,740	68,441	71,013
30………	32,226	35,980	40,021	43,697	47,064	50,337	53,666	56,939	59,888	62,860	65,744	68,496
35………	28,373	32,075	36,090	39,823	43,282	46,682	50,168	53,602	56,727	59,885	62,965	65,910
40………	24,198	27,850	31,843	35,644	39,207	42,750	46,404	50,009	53,321	56,676	59,963	63,112
45………	19,733	23,284	27,207	31,040	34,681	38,348	42,154	45,918	49,425	52,989	56,482	59,834
50………	15,307	18,634	22,367	26,122	29,761	33,482	37,368	41,230	44,894	48,634	52,300	55,828
55………	11,332	14,272	17,648	21,156	24,629	28,248	32,085	35,933	39,649	43,456	47,233	50,877
60………	7,900	10,306	13,155	16,228	19,353	22,688	26,299	29,978	33,603	37,359	41,120	44,781
65………	5,132	6,925	9,125	11,596	14,188	17,031	20,199	23,497	26,824	30,325	33,873	37,364
70………	2,975	4,152	5,654	7,417	9,329	11,495	14,002	16,684	19,466	22,456	25,553	28,654
75………	1,399	2,031	2,876	3,921	5,109	6,511	8,201	10,076	12,102	14,341	16,733	19,185
80………	483	738	1,099	1,574	2,143	2,855	3,757	4,805	5,993	7,354	8,867	10,470
85………	90	150	245	382	558	798	1,125	1,532	2,023	2,616	3,310	4,080
FEMALES												
0………	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
1………	69,324	71,334	73,185	74,899	76,627	78,252	79,675	81,075	82,441	83,799	85,095	86,360
5………	50,372	54,212	57,679	60,823	63,834	66,658	69,144	71,552	73,805	76,041	78,133	80,148
10………	45,703	49,818	53,575	57,004	60,294	63,400	66,151	68,819	71,307	73,783	76,095	78,320
15………	42,805	47,025	50,913	54,486	57,925	61,189	64,095	66,921	69,554	72,182	74,637	77,000
20………	39,519	43,715	47,638	51,289	54,843	58,245	61,300	64,290	67,090	69,900	72,540	75,091
25………	35,771	39,851	43,731	47,399	51,044	54,574	57,763	60,895	63,905	66,933	69,792	72,569
30………	31,724	35,699	39,552	43,257	46,994	50,656	54,003	57,300	60,539	63,809	66,929	69,943
35………	27,532	31,417	35,258	39,018	42,860	46,671	50,189	53,664	57,137	60,655	64,013	67,263
40………	23,316	27,109	30,940	34,757	38,700	42,660	46,348	49,998	53,698	57,458	61,041	64,515
45………	19,344	23,006	26,781	30,613	34,615	38,686	42,507	46,299	50,180	54,139	57,919	61,594
50………	15,648	19,093	22,723	26,481	30,452	34,547	38,437	42,313	46,318	50,421	54,363	58,211
55………	12,222	15,324	18,680	22,235	26,067	30,084	33,945	37,822	41,885	46,075	50,107	54,083
60………	9,134	11,769	14,709	17,911	21,451	25,239	28,937	32,696	36,706	40,883	44,942	48,979
65………	6,324	8,380	10,753	13,423	16,474	19,824	23,162	26,616	30,392	34,381	38,331	42,319
70………	3,920	5,344	7,052	9,046	11,420	14,110	16,860	19,775	23,067	26,614	30,205	33,895
75………	2,003	2,824	3,854	5,110	6,676	8,519	10,476	12,619	15,129	17,905	20,805	23,861
80………	743	1,096	1,566	2,171	2,970	3,969	5,076	6,341	7,898	9,684	11,627	13,743
85………	156	249	385	575	849	1,221	1,660	2,192	2,895	3,744	4,720	5,835

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[*] Equivalent values of $^{o}e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

(l_x) OF MODEL LIFE TABLES

reference in years) [a]

Level 60 (°e₀ = *50*)	*Level 65* (°e₀ = *52.5*)	*Level 70* (°e₀ = *55*)	*Level 75* (°e₀ = *57.6*)	*Level 80* (°e₀ = *60.4*)	*Level 85* (°e₀ = *63.2*)	*Level 90* (°e₀ = *65.8*)	*Level 95* (°e₀ = *68.2*)	*Level 100* (°e₀ = *70.2*)	*Level 105* (°e₀ = *71.7*)	*Level 110* (°e₀ = *73.0*)	*Level 115* (°e₀ = *73.9*)	*Sex and age (x) in years*
												MALES
100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0
85,622	86,918	88,189	89,766	91,553	93,340	94,948	96,199	97,033	97,589	97,959	98,206	1
80,180	82,139	84,025	86,186	88,513	90,840	92,996	94,799	96,087	96,950	97,526	97,911	5
78,599	80,740	82,792	85,110	87,579	90,045	92,326	94,256	95,697	96,679	97,335	97,774	10
77,546	79,799	81,949	84,360	86,913	89,462	91,824	93,838	95,380	96,451	97,167	97,646	15
75,900	78,304	80,598	83,146	85,831	88,510	90,999	93,142	94,842	96,064	96,882	97,430	20
73,575	76,188	78,672	81,409	84,291	87,173	89,860	92,204	94,136	95,548	96,495	97,131	25
71,240	74,055	76,741	79,679	82,744	85,814	88,704	91,251	93,399	94,984	96,050	96,767	30
68,846	71,866	74,756	77,888	81,142	84,399	87,475	90,212	92,554	94,306	95,487	96,282	35
66,247	69,474	72,567	75,894	79,336	82,777	86,038	88,963	91,490	93,413	94,712	95,587	40
63,172	66,614	69,913	73,438	77,073	80,704	84,143	87,258	89,970	92,077	93,528	94,507	45
59,345	62,979	66,511	70,180	73,990	77,799	81,408	84,714	87,610	89,932	91,596	92,748	50
54,521	58,301	61,989	65,811	69,757	73,703	77,454	80,930	84,008	86,548	88,468	89,872	55
48,466	52,315	56,084	59,985	64,008	68,039	71,880	75,462	78,710	81,475	83,639	85,330	60
40,908	44,659	48,373	52,218	56,180	60,158	63,986	67,606	70,965	73,930	76,351	78,320	65
31,847	35,282	38,722	42,291	46,000	49,739	53,384	56,893	60,227	63,290	65,875	68,070	70
21,751	24,567	27,449	30,466	33,638	36,875	40,080	43,227	46,284	49,192	51,732	53,953	75
12,198	14,149	16,183	18,339	20,646	23,036	25,456	27,882	30,322	32,734	34,971	36,999	80
4,943	5,957	7,057	8,255	9,569	10,966	12,415	13,914	15,458	17,015	18,581	20,119	85
												FEMALES
100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0
87,625	88,846	90,042	91,401	92,941	94,481	95,867	96,945	97,663	98,142	98,461	98,674	1
82,136	84,055	85,945	87,890	90,044	92,191	94,139	95,755	96,860	97,600	98,095	98,425	5
80,515	82,636	84,708	86,833	89,160	91,475	93,585	95,350	96,568	97,397	97,952	98,322	10
79,333	81,591	83,792	86,047	88,500	90,939	93,161	95,030	96,328	97,223	97,823	98,223	15
77,615	80,059	82,433	84,865	87,488	90,099	92,486	94,509	95,926	96,932	97,608	98,059	20
75,332	78,010	80,615	83,284	86,128	88,963	91,571	93,806	95,397	96,543	97,314	97,829	25
72,940	75,864	78,702	81,611	84,686	87,758	90,575	93,011	94,786	96,077	96,946	97,528	30
70,500	73,663	76,726	79,861	83,158	86,450	89,465	92,099	94,048	95,489	96,461	97,113	35
67,981	71,374	74,652	78,006	81,511	85,007	88,203	91,023	93,136	94,726	95,800	96,522	40
65,266	68,868	72,342	75,895	79,586	83,266	86,637	89,639	91,908	93,638	94,826	95,625	45
62,064	65,852	69,506	73,244	77,105	80,959	84,490	87,668	90,089	91,958	93,282	94,192	50
58,093	62,044	65,857	69,763	73,785	77,809	81,502	84,862	87,434	89,461	90,912	91,946	55
53,076	57,135	61,080	65,138	69,302	73,479	77,353	80,918	83,642	85,813	87,381	88,509	60
46,413	50,489	54,486	58,627	62,887	67,188	71,196	74,943	77,803	80,097	81,840	83,109	65
37,747	41,630	45,509	49,576	53,770	58,044	62,064	65,892	68,831	71,247	73,184	74,711	70
27,141	30,521	33,962	37,628	41,441	45,385	49,147	52,811	55,662	58,138	60,278	62,094	75
16,086	18,569	21,190	24,054	27,082	30,281	33,399	36,517	38,977	41,193	43,337	45,319	80
7,123	8,541	10,090	11,836	13,726	15,781	17,838	19,966	21,690	23,298	24,963	26,615	85

TABLE IV. SURVIVORS WITHIN AGE

Mortality level (or time-

Sex and age (x) in years	Level 0 ($^{\circ}e_0 = 20$)	Level 5 ($^{\circ}e_0 = 22.5$)	Level 10 ($^{\circ}e_0 = 25$)	Level 15 ($^{\circ}e_0 = 27.5$)	Level 20 ($^{\circ}e_0 = 30$)	Level 25 ($^{\circ}e_0 = 32.5$)	Level 30 ($^{\circ}e_0 = 35$)	Level 35 ($^{\circ}e_0 = 37.5$)	Level 40 ($^{\circ}e_0 = 40$)	Level 45 ($^{\circ}e_0 = 42.5$)	Level 50 ($^{\circ}e_0 = 45$)	Level 55 ($^{\circ}e_0 = 47.5$)
MALES												
T_0	1,980,306	2,214,814	2,475,323	2,722,611	2,957,495	3,194,138	3,441,523	3,689,987	3,924,285	4,165,152	4,404,679	4,637,355
0–4.......	304,577	316,722	331,076	342,668	352,891	362,419	371,985	381,293	389,437	397,482	405,362	412,803
5–9.......	233,498	251,882	271,472	288,018	302,680	316,410	329,868	342,915	354,305	365,595	376,470	386,740
10–14......	216,418	235,728	256,150	273,650	289,255	303,950	318,340	332,322	344,548	356,702	368,380	379,418
15–19......	203,065	222,610	243,295	261,262	277,385	292,665	307,698	322,362	335,225	348,060	360,382	372,048
20–24......	187,505	206,828	227,390	245,562	261,968	277,662	293,312	308,645	322,210	335,808	348,895	361,328
25–29......	170,115	189,058	209,368	227,662	244,320	260,428	276,725	292,732	307,072	321,500	335,462	348,772
30–34......	151,498	170,138	190,278	208,800	225,865	242,548	259,585	276,352	291,538	306,862	321,772	336,015
35–39......	131,428	149,812	169,832	188,668	206,222	223,580	241,430	259,028	275,120	291,402	307,320	322,555
40–44......	109,828	127,835	147,625	166,710	184,720	202,745	221,395	239,818	256,865	274,162	291,112	307,365
45–49......	87,600	104,795	123,935	142,905	161,105	179,575	198,805	217,870	235,798	254,058	271,955	289,155
50–54......	66,598	82,265	100,038	118,195	135,975	154,325	173,632	192,908	211,358	230,225	248,832	266,762
55–59......	48,080	61,445	77,008	93,460	109,955	127,340	145,960	164,778	183,130	202,038	220,882	239,145
60–64......	32,580	43,078	55,700	69,560	83,852	99,298	116,245	133,688	151,068	169,210	187,482	205,362
65–69......	20,268	27,692	36,948	47,532	58,792	71,315	85,502	100,452	115,725	131,952	148,565	165,045
70–74......	10,935	15,458	21,325	28,345	36,095	45,015	55,508	66,900	78,920	91,992	105,715	119,598
75–79......	4,705	6,922	9,938	13,738	18,130	23,415	29,895	37,202	45,238	54,238	64,000	74,138
80–84......	1,432	2,220	3,360	4,890	6,752	9,132	12,205	15,842	20,040	24,925	30,442	36,375
85+.......	176	326	585	986	1,533	2,316	3,433	4,880	6,688	8,941	11,651	14,731
FEMALES												
T_0	2,019,694	2,271,979	2,524,846	2,777,294	3,040,235	3,307,683	3,559,943	3,813,632	4,074,257	4,343,703	4,605,783	4.866,027
0–4.......	314,490	327,880	340,066	351,211	362,113	372,350	381,341	390,108	398,459	406,753	414,581	422,165
5–9.......	240,188	260,075	278,135	294,568	310,320	325,145	338,238	350,928	362,780	374,560	385,570	396,170
10–14......	221,270	242,108	261,220	278,725	295,548	311,472	325,615	339,350	352,152	364,912	376,830	388,300
15–19......	205,810	226,850	246,378	264,438	281,920	298,585	313,488	328,028	341,610	355,205	367,942	380,228
20–24......	188,225	208,915	228,422	246,720	264,718	282,048	297,658	312,962	327,488	342,082	355,830	369,150
25–29......	168,738	188,875	208,208	226,640	245,095	263,075	279,415	295,488	311,110	326,855	341,802	356,280
30–34......	148,140	167,790	187,025	205,688	224,635	243,318	260,480	277,410	294,190	311,160	327,355	343,015
35–39......	127,120	146,315	165,495	184,438	203,900	223,328	241,342	259,155	277,088	295,282	312,635	329,445
40–44......	106,650	125,288	144,302	163,425	183,288	203,365	222,138	240,742	259,695	278,992	297,400	315,272
45–49......	87,480	105,248	123,760	142,735	162,668	183,082	202,360	221,530	241,245	261,400	280,705	299,512
50–54......	69,675	86,042	103,508	121,790	141,298	161,578	180,955	200,338	220,508	241,240	261,175	280,735
55–59......	53,390	67,732	83,472	100,365	118,795	138,308	157,205	176,295	196,478	217,395	237,622	257,655
60–64......	38,645	50,372	63,655	78,335	94,812	112,658	130,248	148,280	167,745	188,160	208,182	228,245
65–69......	25,610	34,310	44,512	56,172	69,735	84,835	100,055	115,978	133,648	152,488	171,340	190,535
70–74......	14,808	20,420	27,265	35,390	45,240	56,572	68,340	80,985	95,490	111,298	127,525	144,390
75–79......	6,865	9,800	13,550	18,202	24,115	31,220	38,880	47,400	57,568	68,972	81,080	94,010
80–84......	2,248	3,362	4,878	6,865	9,548	12,975	16,840	21,332	26,982	33,570	40,868	48,945
85+.......	342	597	995	1,587	2,487	3,769	5,345	7,323	10,021	13,379	17,341	24,975

[a] Equivalent values of $^{\circ}e_0$ shown in parentheses, refer to expectation of life at birth, for both sexes, in years.

GROUPS (L_x) OF MODEL LIFE TABLES

reference in years) [a]

Level 60 ($^{o}e_0$ = *50*)	*Level 65* ($^{o}e_0$ = *52.5*)	*Level 70* ($^{o}e_0$ = *55*)	*Level 75* ($^{o}e_0$ = *57.6*)	*Level 80* ($^{o}e_0$ = *60.4*)	*Level 85* ($^{o}e_0$ = *63.2*)	*Level 90* ($^{o}e_0$ = *65.8*)	*Level 95* ($^{o}e_0$ = *68.2*)	*Level 100* ($^{o}e_0$ = *70.2*)	*Level 105* ($^{o}e_0$ = *71.7*)	*Level 110* ($^{o}e_0$ = *73.0*)	*Level 115* ($^{o}e_0$ = *73.9*)	*Sex and age (x) in years*
												MALES
4,872,148	5,117,013	5,356,512	5,613,376	5,882,432	6,153,098	6,411,344	6,647,631	6,855,509	7,027,221	7,160,427	7,264,597	.......T_o
420,276	427,825	435,153	443,871	453,493	463,115	471,904	479,005	483,920	487,206	489,396	490,859	0–4
396,948	407,198	417,042	428,240	440,230	452,212	463,305	472,638	479,460	484,072	487,152	489,212	5–9
390,362	401,348	411,852	423,675	436,230	448,768	460,375	470,235	477,692	482,825	486,255	488,550	10–14
383,615	395,258	406,368	418,765	431,860	444,930	457,058	467,450	475,555	481,288	485,122	487,690	15–19
373,688	386,230	398,175	411,388	425,305	439,208	452,148	463,365	472,445	479,030	483,442	486,402	20–24
362,038	375,608	388,532	402,720	417,588	432,468	446,410	458,638	468,838	476,330	481,362	484,745	25–29
350,215	364,802	378,742	393,918	409,715	425,532	440,448	453,658	464,882	473,225	478,842	482,622	30–34
337,732	353,350	368,308	384,455	401,195	417,940	433,782	447,938	460,110	469,298	475,498	479,672	35–39
323,548	340,220	356,200	373,330	391,022	408,702	425,452	440,552	453,650	463,725	470,600	475,235	40–44
306,292	323,982	341,060	359,045	377,658	396,258	413,878	429,930	443,950	455,022	462,810	468,138	45–49
284,665	303,200	321,250	339,978	359,368	378,755	397,155	414,110	429,045	441,200	450,160	456,550	50–54
257,468	276,540	295,182	314,490	334,412	354,355	373,335	390,980	406,795	420,058	430,268	438,005	55–59
223,435	242,435	261,142	280,508	300,470	320,492	339,665	357,670	374,188	388,512	399,975	409,125	60–64
181,888	199,852	217,738	236,272	255,450	274,742	293,425	311,248	327,980	343,050	355,565	365,975	65–69
133,995	149,622	165,428	181,892	199,095	216,535	233,660	250,300	266,278	281,205	294,018	305,058	70–74
84,872	96,790	109,080	122,012	135,710	149,778	163,840	177,772	191,515	204,815	216,758	227,380	75–79
42,852	50,265	58,100	66,485	75,538	85,005	94,678	104,490	114,450	124,372	133,880	142,795	80–84
18,259	22,488	27,160	32,332	38,093	44,303	50,826	57,652	64,756	71,988	79,324	86,584	......85+
												FEMALES
5,130,792	5,393,553	5,652,382	5,921,457	6,204,598	6,490,665	6,756,688	6,998,544	7,180,317	7,323,693	7,434,057	7,519,058	.......T_o
429,692	436,957	444,096	451,782	460,386	468,976	476,739	482,990	487,213	490,036	491,921	493,179	0–4
406,628	416,728	426,632	436,808	448,010	459,165	469,310	477,762	483,570	487,492	490,118	491,868	5–9
399,620	410,568	421,250	432,200	444,150	456,035	466,865	475,950	482,240	486,550	489,438	491,362	10–14
392,370	404,125	415,562	427,280	439,970	452,595	464,118	473,848	480,635	485,388	488,578	490,705	15–19
382,368	395,172	407,620	420,372	434,040	447,655	460,142	470,788	478,308	483,688	487,305	489,720	20–24
370,680	384,685	398,292	412,238	427,035	441,802	455,365	467,042	475,458	481,550	485,650	488,392	25–29
358,600	373,818	388,570	403,680	419,810	435,520	450,100	462,775	472,085	478,915	483,518	486,602	30–34
346,202	362,592	378,445	394,668	411,672	428,642	444,170	457,805	467,960	475,538	480,652	484,088	35–39
333,118	350,605	367,485	384,752	402,742	420,682	437,100	451,655	462,610	470,910	476,565	480,368	40–44
318,325	336,800	354,620	372,848	391,728	410,562	427,818	443,268	454,992	463,990	470,270	474,542	45–49
300,392	319,740	338,408	357,518	377,275	396,920	414,980	431,325	443,808	453,548	460,485	465,345	50–54
277,922	297,948	317,342	337,252	357,718	378,220	397,138	414,450	427,690	438,185	445,732	451,138	55–59
248,722	269,060	288,915	309,412	330,472	351,668	371,372	389,652	403,612	414,775	423,052	429,045	60–64
210,400	230,298	249,988	270,508	291,642	313,080	333,150	352,088	366,585	378,360	387,560	394,550	65–69
162,220	180,378	198,678	218,010	238,028	258,572	278,028	296,758	311,232	323,462	333,655	342,012	70–74
108,068	122,725	137,880	154,205	171,308	189,165	206,365	223,320	236,598	248,328	259,038	268,532	75–79
58,022	67,775	78,200	89,725	102,020	115,155	128,092	141,208	151,668	161,228	170,750	179,835	80–84
27,443	33,579	40,399	48,199	56,792	66,251	75,836	85,860	94,053	101,750	109,770	117,775	......85+

TABLE V. SURVIVAL RATIOS (P_x)

Mortality level (or time-

[a] *Sex and age (x) in years*	*Level 0* ($\mathring{e}_0$ = 20)	*Level 5* ($\mathring{e}_0$ = 22.5)	*Level 10* ($\mathring{e}_0$ = 25)	*Level 15* ($\mathring{e}_0$ = 27.5)	*Level 20* ($\mathring{e}_0$ = 30)	*Level 25* ($\mathring{e}_0$ = 32.5)	*Level 30* ($\mathring{e}_0$ = 35)	*Level 35* ($\mathring{e}_0$ = 37.5)	*Level 40* ($\mathring{e}_0$ = 40)	*Level 45* ($\mathring{e}_0$ = 42.5)	*Level 50* ($\mathring{e}_0$ = 45)	*Level 55* ($\mathring{e}_0$ = 47.5)
MALES												
(Births)....	(0.6092)	(0.6334)	(0.6622)	(0.6853)	(0.7058)	(0.7248)	(0.7440)	(0.7626)	(0.7789)	(0.7950)	(0.8107)	(0.8256)
0–4.......	0.7666	0.7953	0.8200	0.8405	0.8577	0.8731	0.8868	0.8993	0.9098	0.9198	0.9287	0.9369
5–9.......	0.9269	0.9359	0.9436	0.9501	0.9556	0.9606	0.9651	0.9691	0.9725	0.9757	0.9785	0.9811
10–14......	0.9383	0.9444	0.9498	0.9547	0.9590	0.9629	0.9666	0.9700	0.9729	0.9758	0.9783	0.9806
15–19......	0.9234	0.9291	0.9346	0.9399	0.9444	0.9487	0.9532	0.9574	0.9612	0.9648	0.9681	0.9712
20–24......	0.9073	0.9141	0.9207	0.9271	0.9326	0.9379	0.9434	0.9484	0.9530	0.9574	0.9615	0.9653
25–29......	0.8906	0.8999	0.9088	0.9171	0.9245	0.9313	0.9381	0.9440	0.9494	0.9545	0.9592	0.9634
30–34......	0.8675	0.8805	0.8925	0.9036	0.9130	0.9218	0.9301	0.9373	0.9437	0.9496	0.9551	0.9599
35–39......	0.8357	0.8533	0.8692	0.8836	0.8957	0.9068	0.9170	0.9258	0.9336	0.9408	0.9473	0.9529
40–44......	0.7976	0.8198	0.8395	0.8572	0.8722	0.8857	0.8980	0.9085	0.9180	0.9267	0.9342	0.9408
45–49......	0.7603	0.7850	0.8072	0.8271	0.8440	0.8594	0.8734	0.8854	0.8964	0.9062	0.9150	0.9226
50–54......	0.7219	0.7469	0.7698	0.7907	0.8086	0.8251	0.8406	0.8542	0.8664	0.8776	0.8877	0.8965
55–59......	0.6776	0.7011	0.7233	0.7443	0.7626	0.7798	0.7964	0.8113	0.8249	0.8375	0.8488	0.8587
60–64......	0.6221	0.6428	0.6633	0.6833	0.7011	0.7182	0.7355	0.7514	0.7660	0.7798	0.7924	0.8037
65–69......	0.5395	0.5582	0.5772	0.5963	0.6139	0.6312	0.6492	0.6660	0.6820	0.6972	0.7116	0.7246
70–74......	0.4303	0.4478	0.4660	0.4847	0.5023	0.5202	0.5386	0.5561	0.5732	0.5896	0.6054	0.6199
75–79......	0.3044	0.3207	0.3381	0.3559	0.3724	0.3900	0.4083	0.4258	0.4430	0.4595	0.4757	0.4906
(80+)......	(0.1095)	(0.1280)	(0.1483)	(0.1678)	(0.1850)	(0.2023)	(0.2195)	(0.2355)	(0.2502)	(0.2640)	(0.2768)	(0.2882)
FEMALES												
(Births)....	(0.6290)	(0.6558)	(0.6801)	(0.7024)	(0.7242)	(0.7447)	(0.7627)	(0.7802)	(0.7969)	(0.8135)	(0.8292)	(0.8443)
0–4.......	0.7637	0.7932	0.8179	0.8387	0.8570	0.8732	0.8870	0.8996	0.9105	0.9209	0.9300	0.9384
5–9.......	0.9212	0.9309	0.9392	0.9462	0.9524	0.9579	0.9627	0.9670	0.9707	0.9742	0.9773	0.9801
10–14......	0.9301	0.9370	0.9432	0.9487	0.9539	0.9586	0.9628	0.9666	0.9701	0.9734	0.9764	0.9792
15–19......	0.9146	0.9209	0.9271	0.9330	0.9390	0.9446	0.9495	0.9541	0.9587	0.9631	0.9671	0.9709
20–24......	0.8965	0.9041	0.9115	0.9186	0.9259	0.9327	0.9387	0.9442	0.9500	0.9555	0.9606	0.9651
25–29......	0.8779	0.8884	0.8983	0.9076	0.9165	0.9249	0.9322	0.9388	0.9456	0.9520	0.9577	0.9628
30–34......	0.8581	0.8720	0.8849	0.8967	0.9077	0.9178	0.9265	0.9342	0.9419	0.9490	0.9550	0.9604
35–39......	0.8390	0.8563	0.8719	0.8861	0.8989	0.9106	0.9204	0.9289	0.9372	0.9448	0.9513	0.9570
40–44......	0.8203	0.8400	0.8576	0.8734	0.8875	0.9003	0.9110	0.9202	0.9290	0.9369	0.9439	0.9500
45–49......	0.7965	0.8175	0.8364	0.8533	0.8686	0.8825	0.8942	0.9043	0.9140	0.9229	0.9304	0.9373
50–54......	0.7663	0.7872	0.8064	0.8241	0.8407	0.8560	0.8688	0.8800	0.8910	0.9012	0.9098	0.9178
55–59......	0.7238	0.7437	0.7626	0.7805	0.7981	0.8145	0.8285	0.8411	0.8538	0.8655	0.8761	0.8859
60–64......	0.6627	0.6811	0.6993	0.7171	0.7355	0.7530	0.7682	0.7822	0.7967	0.8104	0.8230	0.8348
65–69......	0.5782	0.5952	0.6125	0.6300	0.6487	0.6668	0.6830	0.6983	0.7145	0.7299	0.7443	0.7578
70–74......	0.4636	0.4799	0.4970	0.5143	0.5330	0.5519	0.5689	0.5853	0.6029	0.6197	0.6358	0.6511
75–79......	0.3275	0.3431	0.3600	0.3772	0.3959	0.4156	0.4331	0.4500	0.4687	0.4867	0.5040	0.5206
(80+)......	(0.1320)	(0.1508)	(0.1694)	(0.1878)	(0.2066)	(0.2251)	(0.2409)	(0.2556)	(0.2708)	(0.2850)	(0.2979)	(0.3099)

[a] Equivalent values of $\mathring{e}_0$ shown in parentheses refer to expectation of life at birth, for both sexes, in years.

OF MODEL LIFE TABLES

reference in years) [a]

Level 60 (°e₀ = *50*)	*Level 65* (°e₀ = *52.5*)	*Level 70* (°e₀ = *55*)	*Level 75* (°e₀ = *57.6*)	*Level 80* (°e₀ = *60.4*)	*Level 85* (°e₀ = *63.2*)	*Level 90* (°e₀ = *65.8*)	*Level 95* (°e₀ = *68.2*)	*Level 100* (°e₀ = *70.2*)	*Level 105* (°e₀ = *71.7*)	*Level 110* (°e₀ = *73.0*)	*Level 115* (°e₀ = *73.9*)	*Sex and age (x) in years*
												MALES
(0.8406)	(0.8557)	(0.8703)	(0.8877)	(0.9070)	(0.9262)	(0.9438)	(0.9580)	(0.9678)	(0.9744)	(0.9788)	(0.9817)	....(Births)
0.9445	0.9518	0.9584	0.9648	0.9708	0.9765	0.9818	0.9867	0.9908	0.9936	0.9954	0.9966	0–4
0.9834	0.9856	0.9876	0.9893	0.9909	0.9924	0.9937	0.9949	0.9963	0.9974	0.9982	0.9986	5–9
0.9827	0.9848	0.9867	0.9884	0.9900	0.9914	0.9928	0.9941	0.9955	0.9968	0.9977	0.9982	10–14
0.9741	0.9772	0.9798	0.9824	0.9848	0.9871	0.9893	0.9913	0.9935	0.9953	0.9965	0.9974	15–19
0.9688	0.9725	0.9758	0.9789	0.9819	0.9847	0.9873	0.9898	0.9924	0.9944	0.9957	0.9966	20–24
0.9673	0.9712	0.9748	0.9781	0.9811	0.9840	0.9866	0.9891	0.9916	0.9935	0.9948	0.9956	25–29
0.9644	0.9686	0.9725	0.9760	0.9792	0.9822	0.9849	0.9874	0.9897	0.9917	0.9930	0.9939	30–34
0.9580	0.9628	0.9671	0.9711	0.9746	0.9779	0.9808	0.9835	0.9860	0.9881	0.9897	0.9907	35–39
0.9467	0.9523	0.9575	0.9617	0.9658	0.9696	0.9728	0.9759	0.9786	0.9812	0.9835	0.9851	40–44
0.9294	0.9359	0.9419	0.9469	0.9516	0.9558	0.9596	0.9632	0.9664	0.9696	0.9727	0.9752	45–49
0.9045	0.9121	0.9189	0.9250	0.9306	0.9356	0.9400	0.9441	0.9481	0.9521	0.9558	0.9594	50–54
0.8678	0.8767	0.8847	0.8919	0.8985	0.9044	0.9098	0.9148	0.9198	0.9249	0.9296	0.9341	55–59
0.8141	0.8244	0.8338	0.8423	0.8502	0.8573	0.8639	0.8702	0.8765	0.8830	0.8890	0.8945	60–64
0.7367	0.7487	0.7598	0.7698	0.7794	0.7881	0.7963	0.8042	0.8119	0.8197	0.8269	0.8335	65–69
0.6334	0.6469	0.6594	0.6708	0.6816	0.6917	0.7012	0.7102	0.7192	0.7283	0.7372	0.7454	70–74
0.5049	0.5193	0.5326	0.5449	0.5566	0.5675	0.5779	0.5878	0.5976	0.6072	0.6176	0.6280	75–79
(0.2988)	(0.3091)	(0.3186)	(0.3272)	(0.3352)	(0.3426)	(0.3493)	(0.3556)	(0.3613)	(0.3666)	(0.3721)	(0.3775)	.....(80+)
												FEMALES
(0.8594)	(0.8739)	(0.8882)	(0.9036)	(0.9208)	(0.9380)	(0.9535)	(0.9660)	(0.9744)	(0.9801)	(0.9838)	(0.9864)	....(Births)
0.9463	0.9537	0.9607	0.9669	0.9731	0.9791	0.9844	0.9892	0.9925	0.9948	0.9963	0.9973	0–4
0.9828	0.9852	0.9874	0.9895	0.9914	0.9932	0.9948	0.9962	0.9972	0.9981	0.9986	0.9990	5–9
0.9819	0.9843	0.9865	0.9886	0.9906	0.9925	0.9941	0.9956	0.9967	0.9976	0.9982	0.9987	10–14
0.9745	0.9778	0.9809	0.9838	0.9865	0.9891	0.9914	0.9935	0.9952	0.9965	0.9974	0.9980	15–19
0.9694	0.9735	0.9771	0.9807	0.9839	0.9869	0.9896	0.9920	0.9940	0.9956	0.9966	0.9973	20–24
0.9674	0.9718	0.9756	0.9792	0.9826	0.9858	0.9884	0.9909	0.9929	0.9945	0.9956	0.9963	25–29
0.9654	0.9700	0.9739	0.9777	0.9811	0.9842	0.9868	0.9893	0.9913	0.9929	0.9941	0.9948	30–34
0.9622	0.9669	0.9710	0.9749	0.9783	0.9814	0.9841	0.9866	0.9886	0.9903	0.9915	0.9923	35–39
0.9556	0.9606	0.9650	0.9691	0.9727	0.9759	0.9788	0.9814	0.9835	0.9853	0.9868	0.9879	40–44
0.9437	0.9493	0.9543	0.9589	0.9631	0.9668	0.9700	0.9731	0.9754	0.9775	0.9792	0.9806	45–49
0.9252	0.9318	0.9377	0.9433	0.9482	0.9529	0.9570	0.9609	0.9637	0.9661	0.9680	0.9695	50–54
0.8949	0.9030	0.9104	0.9175	0.9238	0.9298	0.9351	0.9402	0.9437	0.9466	0.9491	0.9510	55–59
0.8459	0.8559	0.8653	0.8743	0.8825	0.8903	0.8971	0.9036	0.9083	0.9122	0.9161	0.9196	60–64
0.7710	0.7832	0.7948	0.8059	0.8162	0.8259	0.8345	0.8429	0.8490	0.8549	0.8609	0.8668	65–69
0.6662	0.6804	0.6940	0.7073	0.7197	0.7316	0.7422	0.7525	0.7602	0.7677	0.7764	0.7852	70–74
0.5369	0.5523	0.5672	0.5819	0.5955	0.6088	0.6207	0.6323	0.6410	0.6493	0.6592	0.6697	75–79
(0.3211)	(0.3313)	(0.3406)	(0.3495)	(0.3576)	(0.3652)	(0.3719)	(0.3781)	(0.3828)	(0.3869)	(0.3913)	(0.3957)	.....(80+)

PART II

CHAPTER VI

D. CORRECTING AGE-SEX CENSUS RETURNS

"No census ever taken anywhere in the world has been completely accurate. Errors arise from the following major sources:

(a) The census schedule

(b) Completeness and accuracy with which the enumerators carried out the field work

(c) Data supplied by the respondents

(d) Accuracy of the various office procedures

(e) Accuracy of the printing or other reproduction procedures." 1/

Errors in the published figures on total population and on the sizes of the respective age-sex groups may result from any one of these sources but are usually mainly due to errors of the (b), (c) and (e) types. As a result, the corrections usually required to improve the reported data on age-sex groups deal with two main phenomena; namely, miscount and misstatement of age.

Theoretically, two types of miscount are possible--undercount and overcount. However, it is generally agreed that it is much easier and, hence, much more frequent, that a person is missed from the enumeration than counted twice with the result that, when an error exists in the reported size of an age-sex group it has tended to be in the direction of a net undercount rather than a net overcount. The age group which almost invariably contains an undercount is that "less than 5" years of age (especially less than 1) of both sexes.

1/ A. J. Jaffe, Handbook of Statistical Methods for Demographers, op. cit., p. 85.

In some instances, however, some age-sex groups are reported to be larger than they actually are. This is generally due to the fact that some persons misstate their ages. If such misstatement were a random phenomenon, it would be expected to balance out in most instances. One person overstating his age will likely be compensated for by another understating his age. Unfortunately, however, most instances of misstatement contain a consistent bias. Women, it has been found, tend in many instances to understate their ages while both men and women tend, in some cases, to state their ages in round figures ending with a five or a zero. A misstatement of age results in distortion only in the relevant age-sex group while a miscount affects the accuracy of both the relevant age-sex group and the reported total population.

Evaluating and Adjusting Age-Sex Census Returns

There are a number of methods that may be used in attempting to determine the existence of errors in the reported age-sex census data. 1/ None, however, are applicable to all situations. Moreover, in most instances, only the most obvious discrepancies between the actual and the reported magnitudes of age-sex groups can be detected.

The size of a cohort or age-sex group at any given year [x] is determined by: (1) births for that sex during the birth years of the given cohort; (2) the mortality experience of the cohort from birth to the year x; and (3) the net migration of individuals whose sex and ages correspond to the sex and ages of the given cohort between the years of birth and the year x. It follows, therefore, that if birth, death, and migration data for the relevant period were available and accurate, every age-sex group in the population of the given area may be determined with confidence and any discrepancy between the value thus obtained and the value obtained in the census count may be taken as a reliable estimate of the error contained in the latter. In fact, if birth, death and migration data for the relevant period were available and accurate,

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1/ For a detailed and somewhat complete statement of these methods see, United Nations, Methods of Appraisal of Quality of Basic Data for Population Estimates, ST/SOA/Series A, No. 23, particularly Chapter III.

there would be no need for the census count and the additional expense to the census takers for securing this added information. Unfortunately, while it is true that no census taken anywhere in the world has ever been completely accurate, it is also true that vital and migration statistics are never accurate either. In fact, most countries that have taken population censuses find their census data to be more reliable than their vital statistics records and their migration information. Moreover, demographers interested in studying the populations of sub-national regions generally are unable to find recorded data on sub-national migration.

The relationship between census counts and vital and migration statistics is, nevertheless, valuable in attempting to determine errors in age-sex census data. This is especially true when an independent measure of the degree of inaccuracy of the census statistics is available. For example, as was already mentioned, the cohort "less than 1" year of age is usually considerably understated in the census. If the reported number of births during the year preceding the census is fairly accurate or may be adequately adjusted, a comparison between their number and the number reported to be "less than 1" year of age in the census would reveal the degree of undercount of that age group. More exactly, the number of births may first be multiplied by an appropriate survival ratio ($1 - q_o$, from a relevant life table) and the difference between the result and the reported number of children "less than 1" year of age taken as an estimate of the net undercount of that group. Similarly, adjusted births over the five years preceding the census may be multiplied by an appropriate survival ratio (P_b in the relevant life table) and compared with the census age group "less than 5" years to determine the net undercount in that age group. Both of these procedures, however, assume no migration for the relevant cohort of births. When dealing with the age group "less than 1 year", this may be a realistic assumption even in countries or area that are known to have a substantial over-all rate of net migration because net migration may still be considered negligible for a cohort of births of one year. However, migration presents a more serious problem when the age group being dealt with is "less than 5" years of age.

Estimating Census Undercount for the Age Group "less than 1" Year of Age

The 1956 Census of the Fiji Islands, taken near the end of September of that year, reported a male population, "less than 1" year of age, of 7,201 persons. The degree of undercount in this figure may be obtained as follows: [1]

The relevant birth period is approximately the first ten months in 1956 and the last two months in 1955. Male births in 1956 and 1955 were reported at 7,205 and 6,741 respectively. The number of reported male births over the relevant period may, therefore, be obtained thusly:

$$\text{Reported births (November 1955 through October 1956)} =$$

$$2/12\ (6,741) + 10/12\ (7,205) = 7,128$$

Assume, for the sake of illustration that, through an independent and reliable test, [2] it is found that the completeness of registration of births is only 0.85 or 85 percent of actual births. In order to obtain actual births, one may use the following formula:

$$\text{Actual births} = \text{Recorded births} \times \frac{1}{\text{Completeness of registration}}$$

$$\text{hence: } 7128 \times \frac{1}{0.85} = 8,386, \text{ actual male births (November 1955 through October 1956).}$$

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[1] The following is only for illustration purposes and should not be taken as an actual estimate of undercount in the Fiji Islands 1956 Census.

[2] See Section E, Chapter VI.

Once the actual number of births is estimated, it is necessary to find the expected number of male "less than 1" year of age in the Fiji Islands at the end of November, 1956. This may be obtained by multiplying the number of actual male births by an appropriate survival ratio. From the Fiji Islands male life table we obtain the life table mortality rate, $q_o = 0.0583$ (see Table XIII, Section C, Chapter VI) and by subtracting it from 1.0000 we obtain the relevant survival ratio. [1] Hence:

$$1 - q_o = 1.0000 - 0.0583 = 0.9417$$

The expected number of males "Less than 1" year of age on the 1956 census date is therefore:

Expected number "less than 1" = Actual births x survival ratio

or:

$$8386 \times 0.9417 = 7897$$

The final step; namely, the estimation of the net undercount of males "less than 1" year of age, is effected by subtracting the reported from the expected number. The number of males "less than 1" year of age in the 1956 census is 7,201. Hence:

Net Undercount of "less than 1" = 7,897 - 7,201 = 696 persons.

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[1] The use of $1-q_o$ is over-simplified in that it is the probability of surviving for 12 months while the probability of persons born between November 1955 through October 1956 of surviving until the census month (November 1956) is for only six months (assuming one-half are born before May 1, 1956 and one-half are born after that date). A rough approximation of the mortality to which this population is exposed (assuming that about 60 percent of all infant deaths occur during the first month of life) is about 80 percent of the survival ratio for the entire year (q_o). See W. P. D. Logan, "The Measurement of Infant Mortality," U. N. Population Bulletin No. 3, October, 1953.

To summarize:

(1): Recorded births = 7,128

(2): Adjusted births = 8,386

(3): Life table mortality rate q_o = 0.0583

(4): [1.0000 - (3)] Survival ratio = 0.9417

(5): [(2) x (4)] Expected number "less than 1" = 7,897

(6): Reported number "less than 1" = 7,201

(7): [(5) - (6)] Undercount 696

Estimating Census Undercount for the Age Group "less than 5" Years of Age

The procedure in this instance is very similar to the one just described and, hence, requires little attention. The country chosen for illustration purposes is Thailand and the census chosen is that taken in the middle of 19[illegible]

Recorded male births during the relevant period (i.e., middle of 1942 through the middle of 1947) were 1,387,300. It is again assumed that the completeness of registration of males is known to be approximately 0.85.

Therefore:

$$1,387,300 \times \frac{1}{0.85} = 1,632,100$$

The relevant survival ratio is the P_b rate from the life table for males around 1945 (which corresponds to the P_b function of column 7 of Table XIII Section B, Chapter VI). This was computed to be approximately 0.8987.

The expected number of males "Less than 5" years of age in the middle of 1947 is, therefore:

$$1,632,100 \times 0.8987 = 1,466,800$$

The number of males "Less than 5" years of age actually enumerated, however, was 1, 328, 600.

The net undercount for that age-sex group is, therefore:

1, 466, 800 - 1, 328, 600 = 138, 200 persons.

Adjusting Five-Year Age Groups Ten and Over

As we have already mentioned, the reported size of five-year age groups other than the first may also contain errors. Such errors, however, are less frequent and generally relatively smaller than the error in the reported size of the age group "Less than 5". Furthermore, these errors are usually in the nature of a net misstatement of age rather than in the nature of a net undercount. 1/

A close look at the age distribution in a census may show that one five-year age group is reported distinctly larger than one or more of the younger five-year age groups. This, however, may be due to a drastic fluctuation in the past birth rates and/or to different age-sex specific migration rates. If there is no reason to believe that such factors are relevant in the particular case, an error in reporting should be suspected and adjustment in the reported size of the given age group becomes in order.

1/ The illustrated methods of evaluation and adjusting census returns of five year age-sex groups are by no means exhaustive. They were all chosen because of their simplicity and relative reliability. For other methods consult A. J. Jaffe, Handbook of Statistical Methods for Demographers, op. cit., pp. 11-15; and United Nations, Methods of Appraisal of Quality of Basic Data for Population Estimates, ST/SOA/Series A, Population Studies No. 23, pp. 31-54.

In other instances, a comparison between the reported size of a given age group of males with the same age group of females shows a distinctly high ratio of one group to the other. In such instances, past birth rates are not usually the cause of such discrepancies for the ratio of male to female births rarely changes--at least not suddenly. Such discrepancies may arise, however, from differential age-sex migration rates. If no such factor is believed to be relevant in the situation at hand, it may be concluded that at least the size of one age-sex group contains an error. Adjustment may then be deemed necessary.

If the age-sex distribution in two censuses is compared, one may also detect errors in census counts. For example, assume that the two censuses in question are five years apart, and that, as in Part II, Chapter II, (Worksheet 5) each age-sex group is multiplied by an appropriate survival ratio and the result is compared with the relevant age-sex groups in the second census thus obtaining net migration by age and sex (column 6 of same worksheet). It may be found that one or two age-sex groups show either a different sign of migration than the age-sex groups immediately before and after, or that males of given ages are migrating in a different direction than females. These findings may very well be what actually took place. However, if there is no reason to believe that this is so, one may suspect an error in the reported size of one or more of the relevant age-sex groups.

One method of adjusting errors of this nature is that of applying a "smoothing" formula to the relevant data. To illustrate this, the example of the Fiji Islands can be used. In Worksheet 21 (below) net migration is obtained for selected male age groups between 1946 and 1956. The method used is the same as in Worksheet 5 of Part II, Chapter II referred to above.

It is noted that all age-sex groups considered show net outmigration except the center age-sex group--namely, "40 through 44" in 1956 or "30 through 34" in 1946. If the investigator finds no reason to believe that this cohort, on balance, actually outmigrated in this high proportion, an error in the reported size of the male age group "30 through 34" in 1946 and/or the male age group "40 through 44" in 1956, is indicated. For illustrative purposes, both will be corrected.

If the size of the relevant five year age-sex group is designated by L_o, that of the immediately younger by L_{-1} and that of the immediately older by L_1, a smoothing formula of the following form may be used to adjust the reported size L_o of the age-sex group in question:

$$\text{Adjusted } L_o = \frac{L_{-1} + 2L_o + L_1}{4}$$

Hence, for 1946: Adjusted size of "30 through 34" =

$$\frac{9907 + 2(7,960) + 6,624}{4} = 8,113$$

and for 1956: Adjusted size of "40 through 44" =

$$\frac{9,299 + 2(7,775) + 5,960}{4} = 7,702$$

If these two adjusted figures are used, Worksheet 22 will be substituted for Worksheet 21. Note that the relevant cohort still shows net outmigration for the decade 1946-1956; but the magnitude of this outmigration has been reduced considerably.

In case of an open-end listing of age groups, the smoothing method just described may not be applied to the oldest two age brackets. This, however, is not a serious deficiency for the size of these age groups is usually relatively small compared to the younger groups so that, in general, even a considerable error in their reported sizes would not greatly affect the total migration estimate. 1/

1/ For methods of adjusting the oldest two age groups of an open-end listing, see the two references mentioned in the previous footnote.

WORKSHEET 21

ESTIMATED MIGRATION FOR SELECTED AGE GROUPS OF MALES, THE FIJI ISLANDS, 1946-1956

Age Group	Population 1946 Census	10-year Survival Ratio	Population Excluding Migration 1956	Population 1956 Census	Net Migration 1946-1956
(1)	(2)	(3)	(4)	(5)	(6)
20 through 24	10,776	0.9670	-	-	-
25 " 29	9,907	0.9579	-	-	-
30 " 34	7,960	0.9459	10,420	10,167	-253
35 " 39	6,624	0.9252	9,490	9,299	-191
40 " 44	4,928	0.8929	7,529	7,775	+246
45 " 49	-	-	6,129	5,960	-169
50 " 54	-	-	4,400	4,285	-115

WORKSHEET 22

ESTIMATED MIGRATION FOR SELECTED AGE GROUPS OF MALES (WITH ADJUSTMENT) THE FIJI ISLANDS, 1946-1956

Age Group	Population 1946 Census	10 year Survival Ratio	Population Excluding Migration 1956	Population 1956 Census	Net Migration 1946-1956
(1)	(2)	(3)	(4)	(5)	(6)
20 through 24	10,776	0.9670	-	-	-
25 " 29	9,907	0.9579	-	-	-
30 " 34	8,113 1/	0.9459	10,420	10,167	-253
35 " 39	6,624	0.9252	9,490	9,299	-191
40 " 44	4,928	0.8929	7,674	7,702 2/	+ 28
45 " 49	-	-	6,129	5,960	-169
50 " 54	-	-	4,400	4,285	-115

1/ Adjusted (see formula in text)

2/ Adjusted (see formula in text)

PART II

CHAPTER VI

E. ADJUSTING BIRTH AND DEATH DATA [1]

"The accuracy of the reporting or registration of vital statistics is a function of two components: (a) the degree to which all events are reported, and (b) the degree to which the information provided about each event is accurate."[2] It is generally true, moreover, that while... "Probably no vital statistics reporting system now in existence is completely accurate with respect to both of these phases," [3] death statistics are generally more accurate than statistics of birth. Consequently, the main emphasis in this section will be placed on the evaluation and adjustment of reported births.

BIRTHS

"For the purpose of evaluating the completeness of birth reporting, we are concerned exclusively with statistics of live births, for only live births (unlike still births) represent additions to the population. A 'live birth' may be defined as a child showing any evidence of life such as heart action, breathing, or voluntary movement of muscles, after complete birth, i.e., complete emergence of the child from the body of the mother. The length of time that the child remains alive after complete birth is not considered here as a criterion of a 'live birth'. Obtaining such statistics, therefore, involves the problem of differentiating between live and still births. Ordinarily, if the birth occurs either in a hospital or at home with a physician in attendance, there is little, question about differentiating between live and still births. Even here, however, there are occasions where the infant either dies at the moment of birth or within a very few seconds following birth, such that it is difficult to classify it as one or the other. When the birth occurs in a home or elsewhere without a physician in attendance, the probability of mis-classification is greatly increased. Of course, if the infant does live for any appreciable length of time, such as several hours, it should be possible to obtain the fact that it was a live birth regardless of whether or not a physician or nurse was in attendance. In such a case, the length of the gestation period need not be considered since a seven month fetus which manages to survive is just as valid a statistic as a nine month fetus.

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[1] This section draws heavily on Chapter V of A. J. Jaffe's, Handbook of Statistical Methods for Demographers, op. cit.

[2] Ibid., p. 139.

[3] Ibid.

"Once it has been determined that the incident should be classified as a live birth, the next problem is that of obtaining a complete count of all such events. Ordinarily, when there is a doctor in attendance it is relatively easy to obtain a notification or registration of the event. This is not necessarily the case, however, in situations where there is no doctor in attendance, particularly in the event of an illegitimate birth. It is relatively easy to obtain the cooperation of doctors since there are only a limited number of them who have to be familiarized with the requirements of the registration system; then, too, they are generally organized in associations facilitating contacts between them and the vital statistics registrars. It is probably still easier to obtain the cooperation of hospitals since there are only a limited number of such establishments.

"To obtain the registration of births which have occurred without a doctor in attendance poses the much more difficult problem of obtaining the cooperation of a tremendously greater number of individuals, each of whom has only a momentary contact with, and often indifferent interest in, the process of birth registration. The great bulk of the people are certainly much less familiar with the necessities and mechanics of vital registration and perhaps less inclined to register births than is the medical profession. Accordingly, births which occur outside of hospitals and without a doctor in attendance are probably less fully reported. In this connection the midwife perhaps occupies an intermediate position; if the midwives are all licensed and organized, as in England, the births which they attend can be as fully reported as births attended by physicians. In those societies in which stigma is attached to illegitimate births, there may be a deliberate attempt on the part of the mothers, to avoid reporting them, or to report them as legitimate. Of course, to the extent that the latter procedure is followed, the measure of the addition to the population via natality, is not affected (although statistics on marital fertility, may be in error)." 1/

Three methods of estimating the completeness of registration of births will be introduced in this section 2/ --Comparing Recorded Births with Data from other Sources; Name for Name Checking; and the Reverse Survival Ratio Method.

Comparing Recorded Births with Data from other Sources

"The reported total number of births from some outside source can be compared with the total, as reported by the civil [i.e., vital statistics] registrar. Thus, in Brazil the number of baptisms recorded by the Catholic church in 1937 was 1,475,489; in contrast, the number of births reported by the civil registrar was only 527,275. The Catholic church does not claim to have a complete count of all births in Brazil; nevertheless, its figure is more than double that of the number of births reported by the vital statistics registration system. Accordingly, even this very rough comparison indicates that over half of the births in Brazil in 1937 were not reported to the civil registrars." 3/

1/ Ibid.

2/ Other methods may be found in, Ibid. Chapter V; United Nations, Methods of Appraisal of Quality of Basic Data for Population Estimates, op. cit. Chapter II; and the relevant articles in the Bibliography.

3/ A. J. Jaffe, Handbook of Statistical Methods for Demographers, op. cit., p. 144.

Name for Name Checking

"Essentially this method of testing birth registration consists of securing, in some way, a list of children born during a specified period in a certain area and then checking to see whether or not birth certificates were filed for those children.

"Lists of names of children can be secured from a wide variety of sources, including religious registers of baptisms, births reported in newspapers and other publications, and through house-to-house inquiry. The aim is to select a relatively representative list of names for some particular area--(it need not be a complete list). In such a case, the proportion of names for which no birth certificates can be found provides a basis for estimating the degree of under-reporting.

"Perhaps the most elaborate name-for-name checking which has been done, at least in the United States, was the study "Completeness of Birth Registration in the United States, December 1, 1939, to March 31, 1940." This study provided estimates of the completeness of birth registration for each state, county, and city in the United States by color, and also by place of birth, that is, in or outside institutions.

The general procedures were as follows:

> During the census enumeration in 1940, the enumerators were instructed to fill out a special infant card for all infants who were stated to have been born during the period December 1, 1939, to March 31, 1940, inclusive, and who were alive on April 1, 1940. At about the same time all State and certain city bureaus of vital statistics prepared special copies of death certificates filed for children born during this period but dying before April 1, 1940.
>
> The basic problem then was to match each infant card and each copy of a death certificate with a copy of a birth certificate. Each unmatched infant card or death transcript would then be assumed to represent an unregistered birth.

"Essentially, the system is very simple. The job becomes complicated and the procedures elaborate only insofar as the mechanics of matching names becomes quite involved. It will be noted that, in order to carry through such name-for-name checking, it is practically imperative that the vital statistics registration system maintain the records in some form which permits obtaining data for the individual case with a minimum of effort. This means a system whereby the record for each individual birth is kept on a standard form readily accessible to the investigator. This will then permit relatively easy handling of the birth data.

"The general results of this survey indicate that birth registration in the United States during these four months was most nearly complete for those births which occurred in institutions located in urban places of 100,000 population or more, and least complete for those births which occurred in rural areas and outside of institutions. The respective figure for total United States is 92.5 per cent completeness of birth registration.

"It is estimated that the completeness of birth registration had increased to 94.1 per cent by 1944. These estimates were made by assuming that the completeness of registration for births occurring in institutions and those occurring outside institutions remained unchanged after 1940. The number of births occurring in and out of institutions during each year subsequent to 1940 were weighted by these constants to arrive at an overall estimate of under-registration for the United States. The completeness of registration for the total United States increased, then, simply because the proportion of births occurring in institutions increased markedly after 1940. In the earlier year slightly more than half of all births occurred in institutions, as contrasted with over three-quarters of births so occurring in 1944.

"If a nationwide name-for-name check is deemed unfeasible, spot checks can be made to obtain some idea of the degree of completeness of birth registration. The article "Birth Test Surveys, Comparison of Birth Tests by Several Methods in Georgia and Maryland" describes in detail how such local studies were made in the United States during the 1930's. Attention is called to the fact that serious biases can occur in these mail tests where the postal system does not reach all of the population and where there is a large ratio of illiteracy". 1/

The Reverse Survival Ratio Method

In Section D of this chapter, a method of estimating the net undercount of children "Less than 1" year of age and "Less than 5" years of age was discussed. The reader will remember that the procedure was to discount relevant actual births by an appropriate survival ratio to obtain an estimate of the expected numbers in the given age group. This number is then compared with the actual census count to determine the net undercount in the latter. This method, it was pointed out, assumes, among other things, that the number of reported births is fairly accurate or that a reliable correction factor, obtained independently of the census count of children (as, for example, in the two methods just explained), is available so that the reported births may be adjusted accordingly. It stands to reason, therefore, that if the census count of children is accurate or may be adjusted without the use of the relevant birth data, any deficiency in the latter may be estimated by the reverse procedure.

More specifically, to estimate the completeness of birth registration, one may begin with a reliable estimate of the number of children under a certain age for a given census year, "revive" this cohort to their birth period with the use of the inverse survival ratio, and compare the result with the number of recorded births during that period.

But "reviving" a cohort with the use of inverse survival ratios does not take into account the effect of net migration on the size of this cohort. However, if migration is recorded with reasonable accuracy the size of the initial cohort

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1/ Ibid. pp. 144 - 145.

should be adjusted accordingly, reducing it in the case of net in-migration and increasing it in the case of net out-migration in order to obtain a reliable estimate of what the actual size of the cohort would have been if no migration had taken place, i. e., if the births and deaths during the relevant period accounted for all of it. But even in case total net migration is known to be fairly substantial, the number of children "less than 1" will still likely be unaffected to any appreciable extent by it, so that it may be used to estimate the completeness of birth registration during the year preceding the census. In Worksheets 23 and 24 the Reverse Survival Ratio Method is illustrated using the cohorts "less than 1" and "less than 5" respectively obtained from Chile's census count of April 24, 1952.

WORKSHEET 23

ESTIMATING COMPLETENESS OF BIRTH REGISTRATION IN CHILE, APRIL 1951 - APRIL 1952 1/

(1)	:Census Count "Less than 1"	164, 196	
(2)	:Estimated Undercount	62, 065	2/
(3) = (1) + (2)	:"Less than 1" Adjusted	226, 261	
(4)	:Life Table Mortality Rate q_o	0. 1308	3/
(5)=1. 0000-(4)	:Life Table Survival Ratio	0. 8692	4/
(6)=(3) ÷ (5)	:Expected Births April 1951 - April 1952	260, 309	
(7)	:Recorded Births, 1951	191, 332	
(8)	:Recorded Births, 1952	199, 120	
(9)=2/3(7) + 1/3(8)	:Recorded Births April 1951 - April 1952	193, 927	
(10)=(6)-(9)	:Underregistration of Births, April '51 - April '52	66, 382	
(11)=(9) ÷ (6)	:Rate of Birth Registration Completeness	0. 7450	

1/ For illustrative purposes only.

2/ The total reported net undercount was 620, 654 one tenth of which was assumed to apply to the cohort "Less than 1".

3/ Computed by the writer. For Method of Computation see Section C, Chapter VI, Table XIII.

4/ Note that this is 1. 0000 - q_o and not the P_b of the life table which refers to survival for five years of births.

WORKSHEET 24

ESTIMATING COMPLETENESS OF BIRTH REGISTRATION IN CHILE, APRIL 1947 - APRIL 1952 1/

(1)	: Census Count "Less than 5"	779, 139
(2)	: Estimated Undercount	206, 885 2/
(3)=(1) + (2)	: "Less than 5" Adjusted	986, 024
(4)	: Life Table Survival Ratio, P_b	0.8556 3/
(5)=(3) ÷ (4)	: Expected Births, April 1947 - April 1952	1, 152, 435
(6)	: Recorded Births April 1947 - April 1952	949, 505
(7)=(5) - (6)	: Underregistration of Births, April '47-April '52	202, 930
(8)=(6) ÷ (5)	: Rate of Birth Registration Completeness	0.8239

1/ For illustrative purposes only.

2/ The total net undercount was reported at 6, 206, 544, one third of which was assumed to apply to the cohort "Less than 5".

3/ Computed by the writer. For method of Computation see Section C, Chapter VI, Table XIII. Note that the P_b figure is used (rather than $1.0000 - q_o$ as in Worksheet 23).

DEATHS

"The concern here is with statistics about people who were once living and are now dead; thus still births are excluded. The problem of obtaining complete registration of deaths is least acute if deaths are usually attended by physicians or where official burial permits are required before interment. In the first case the doctors can be required to fill out death certificates, and in the second case burial permits can be issued only on the showing of a death certificate, thus insuring the filling out of such a certificate. Where doctors are seldom in attendance or bodies are buried privately without burial permits, the problem of obtaining records of deaths is similar to that of obtaining reports on births which occur without physicians in attendance.

"Rural populations usually have less adequate medical service and live in more out-of-the-way places where burials can be performed more easily without permit. Hence, the completeness of birth and death reporting is usually less in rural than in urban areas." 1/

Only one method of estimating deficiencies in death statistics will be explained. This method consists essentially of a comparison between reported deaths and an estimate thereof obtained with the use of age-sex mortality rates from what seems to be an appropriate mortality "level". It follows, therefore, that three separate steps are required: determining the mortality level to correspond to the United Nations m_x table U. N. I. (see Section C, Chapter VI); multiplying the age-sex mortality rates thus obtained by the respective age-sex groups; and comparing the results with the corresponding recorded death data. Worksheet 25 illustrates this technique using data for the Fiji Islands. 2/

1/ Ibid., p. 139.

2/ Note that this method consists essentially in the reverse procedure of obtaining the m_x values from recorded death data (Worksheets 16 and 17; Section C, Chapter VI) which were then assumed to be accurate.

WORKSHEET 25

ESTIMATING COMPLETENESS OF REGISTRATION OF MALE DEATHS IN FIJI ISLANDS, 1950-1952 1/

Age Group	Model Age-Sex Specific death rates 2/	Population 1951	Expected Deaths 1951	Average Recorded Deaths 1950-52	Deficiency In Death Registration 3/
(1)	(2)	(3)	(4)=(2)x(3)	(5)	(6)=(4)-(5)
Less than 1	0.09018	6,068	547	397	150
1 through 4	0.00845	20,939	177	220	-43
5 " 9	0.00212	23,292	49	42	7
10 " 14	0.00153	19,237	29	26	3
15 " 19	0.00251	15,692	39	38	1
20 " 24	0.00362	12,955	47	37	10
25 " 29	0.00370	11,287	42	38	4
30 " 34	0.00391	9,063	35	38	-3
35 " 39	0.00450	7,962	36	44	-8
40 " 44	0.00579	6,352	37	47	-10

1/ For illustrative purposes. Data shown only through age group 40-44. Data for additional groups may be computed if necessary.

2/ The values in this column may also be obtained from the life table of an area or country believed to have the same mortality conditions as the area under investigation and where recorded deaths on which the life table is based are believed to be accurate.

3/ The minus sign denotes over-registration which may be due to misstatement of age at death, the use of an inappropriate mortality level or to a cenus undercount.

Even when recorded data on deaths is known to be inaccurate, reliable estimates of the number of deaths for a given age-sex group (m_x), or an estimate of life expectancy at birth (^oe_o), may be available through independent sources or through adequate sampling surveys. If such is not available, it becomes necessary to obtain the death data of another area or country where mortality conditions are believed to be approximately the same as in the area under consideration. For the sake of illustration, assume that an independent and reliable estimate of 545 deaths of male children under one year of age is available for the Fiji Islands in 1951, the year of the census. By dividing this number by the census count for that age-sex group (6, 068) the age-sex specific death rate (m_x) for that cohort is obtained. This is equal to 0.8982. From table U.N.I. (Section C, Chapter VI), it is found that this corresponds approximately to mortality level 80 where the corresponding 1000 m_x = 90.18. If this is the appropriate mortality level, and if the mortality conditions in the area conform to the United Nations model, 1/ a list of age-sex specific death rates may be obtained from the column of figures for that level in Table U.N.I. by dividing each percentage in that column by 1000 [Column (2) Worksheet 25].

If each age-sex mortality rate [Column (2)] is multiplied by the corresponding census count for that group [Column (3)], an estimate of deaths for each cohort is obtained [Column (4)].

If recorded deaths for each age-sex group [Column (5)] are subtracted from the corresponding estimated deaths for that group an estimate of the deficiency in recorded deaths is obtained [Column (6)]. 2/ The total of these figures

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1/ The fact that the mortality conditions of the area in question may not correspond to those postulated in the United Nations models constitutes a major shortcoming of this method. However, the only other method, short of an actual count, would be to utilize the mortality experience of a population whose conditions of life were generally similar to those of the population under study, and whose deaths were believed to be more accurately recorded. Fortunately, however, death registration is more apt to be accurately recorded.

2/ In order to minimize random fluctuations, average deaths for each age group over three years centered around the census year are used (as in Worksheet 14, Section B, Chapter VI).

[of Column (6)] may be taken as an estimate of total under-registration for that sex. [The rate of completeness may be obtained by dividing the total of column (5) by the total of column (4)].

The roughness of this method is obvious. It should not, therefore, be used unless death data are known to be extremely deficient. Moreover, it may often be more advisable to use mortality rates of a country believed to have comparable mortality conditions than the "Model" rates [of Column (2)].

PART III

THE LABOR FORCE

ESTIMATING AND PROJECTING THE LABOR FORCE FROM POPULATION DATA

The Labor force or work force is usually defined as the total of employment and unemployment. From the demographic point of view, however, the labor force is viewed as a certain proportion of total population or, more exactly, as a certain ratio of the population of working age. This latter ratio may be designated as the "over-all participation rate." It follows, therefore, that in order to estimate the size of the labor force in a given area, an estimate of total population together with an estimate of the over-all participation rate are essential. Needless to say, the size of the labor force thus obtained should be equal to that obtained by the summation of employment and unemployment. 1/

While the total size of the labor force is usually valuable for an understanding of the manpower problems of a community or nation, it is often more important to obtain the age and sex distribution of this labor force. It is often essential to the planner to know whether the labor force of a community is mainly old or mainly young, whether it is aging or expected to age in the near future or whether a large proportion of it is or will be young new entrants. It may, therefore, be necessary to obtain estimates of the labor force by age and sex.

1/ If, however, employment and unemployment data for the area are collected by place of work of the labor force, their total will not necessarily be equal to the labor force of the census which is by place of residence. The latter figure represents that part of the population of the area that is in the work force irrespective of whether they work in the area or outside it. The labor force by place of work represents employment and unemployment in the area, irrespective of whether the workers reside in the given area or outside of it. The difference between these two figures is equal to net in- or out-commuting.

In order to do this, it is essential to determine the proportion of each age-sex group that is in the labor force; namely, the age-sex participation rates.

This chapter will explain how age-sex participation rates may be obtained from census data, how they may be projected into the future and how a future estimate of the labor force by age and sex may be effected.

CENSUS AGE-SEX PARTICIPATION RATES

Most censuses of population, as noted earlier, list the population of the country by five-year age-sex groups. Most censuses also report the total size of the labor force together with its age-sex breakdown, again by five-year age-sex groups. In such instances, obtaining the age-sex participation rates becomes a simple matter. 1/ In Worksheet 26, computation of the age-sex participation rates for New Caledonia's non-indigenous population is undertaken. The figures in Columns (4) and (7), except the last row (total), are the age-sex participation rates for males and females respectively. The figures in the last row of these two columns refer to the over-all participation rates 2/ of males and females respectively. Note that the latter rates tend to change with every change in the age-sex population distribution even if the age-sex participation rates remain the same. The latter rates tend, therefore, to be more predictable over time.

1/ In many censuses, these participation rates by five-year age-sex groups are published in the same table as the age-sex breakdown of the labor force.

2/ In this instance, this rate refers to the proportion of total population, rather than population of working age, that is actually in the labor force. It would have been more appropriate to eliminate children who could not possibly be in the labor force from the listing of the population age-sex groups. This would change the figures in the first and last rows [except for Columns (3) and (6)].

In some censuses, the age-sex distribution of the labor force [Columns (3) and (6) in Worksheet 26] is reported in "broad" age-sex brackets. In such instances, it may be necessary to combine the population five-year age-sex groups into brackets of the same order and to obtain the age-sex participation rates by "broad" age-sex groups. 1/ The remaining procedure is exactly the same as in Worksheet 26.

1/ In some instances the youngest age-sex bracket for which labor force data reported in the census is not of exactly five years (or a multiple thereof). For example, the youngest group may be "14 through 19" which includes the five-year group "15 through 19" and the number 14 years of age out of the five-year group "10 through 14". In such instances, if no separate count of the persons 14 years of age is reported in the census, an estimate of it may be made through one of the methods described in Chapter V of Part II (such as the one using the Sprague Multipliers). The result should then be added to the population group "15 through 19" to obtain an estimate of the number "14 through 19".

WORKSHEET 26

COMPUTING AGE-SEX PARTICIPATION RATES FOR NEW CALEDONIA, CENSUS OF OCTOBER 9, 1951 NON-INDIGENOUS POPULATION

	MALE			FEMALE		
Age Group	Population	Labor Force	Participation Rates	Population	Labor Force	Participation Rates
(1)	(2)	(3)	(4)=(3)÷(2)	(5)	(6)	(7)=(6)÷(5)
Under 15	3,652	8	0.002	3,460	3	0.001
15 19	944	450	0.477	883	130	0.147
20-24	997	864	0.867	843	190	0.225
25-29	952	841	0.883	761	173	0.227
30-34	745	658	0.883	640	150	0.234
35-39	803	706	0.879	749	176	0.235
40-44	694	597	0.860	641	151	0.236
45-49	568	479	0.843	515	112	0.217
50-54	428	347	0.811	420	90	0.214
55-59	310	236	0.761	314	77	0.245
60-64	263	171	0.650	290	46	0.159
65-69	213	110	0.516	204	23	0.113
70 and over	296	111	0.375	268	32	0.119
Total	10,865	5,578	0.513	9,988	1,353	0.135

PROJECTING AGE-SEX PARTICIPATION RATES

If only one set of age-sex participation rates is available and if the period of projection is not too long, these rates may be held constant to the end of the period. If, on the other hand, sets are available for more than one point in time, the rates for each age-sex group may be extrapolated to the end of the projection period by any of the extrapolation methods explained in Chapter III of Part I in conjunction with the projection of total population.

Because age-sex participation rates, especially for five-year groups, are never completely accurate 1/ and, in some instances, contain considerable error, straight line extrapolation (arithmetical or geometrical) should be undertaken with great caution. The rate for any one age-sex group may thus erroneously show a declining trend over time while actually it is moving upward. It may, therefore, be helpful to the reader to know what the observed trends have been in the more developed economies. 2/ In general the main trends have been as follows:

(1) The participation rates of males under twenty-five years of age have been declining because of the greater emphasis being placed on education.

(2) The participation rates of males over sixty years of age have also been declining because of the decline in the conventional age of retirement.

1/ This inaccuracy may result from errors either in the reported sizes of population age-sex groups or in the reported numbers of persons in the labor force age-sex groups.

2/ The applicability of these trends to a given country hinges upon the extent to which the country in question utilizes the same definitions of the labor force and is experiencing similar socio-economic development.

(3) The participation rates of women, after early childbearing ages, have been increasing because of the increasing social acceptance of women workers.

(4) The participation rates of women under twenty-five and over sixty have been indeterminate because of the downward pull of factors in (1) and (2) above and the upward pull of factors in (3). They have generally, however, been slightly declining.

Therefore, if it is found that in given areas, some age-sex participation rates are reported moving in a different direction than expected, it may be wise to project them either by keeping the most reliable of them constant or by averaging a set of them.

In Worksheet 27, the age-sex participation rates of the New Caledonia (non-indigenous) males are projected to 1956 on the basis of the participation rates of 1946 and 1951. A close look at the data for these two years indicates the probability of understatement in 1951. It will be found, for instance, that for all five-year age-sex groups between thirty and sixty years of age, the participation rates have fallen. These age groups, however, normally remain fairly constant over time so that, aside from random error, it seems probable that this consistent fall indicates a bias of understatement in 1951. 1/ The

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1/ Theoretically, this fall may be due to overstatement of the 1946 rates. However, rates of little over 0.9000 for these age groups are not high enough to warrant such a conclusion and, in fact, seem slightly too low. Besides, the labor force for these ages is usually proportionately more underenumerated than the population so that it is generally improbable that age-sex participation rates are overstated for any given census year.

respective 1956 rates for these age groups will, therefore, be assumed to equal those of 1946. Furthermore, if such an understatement is true, the indicated decline in the participation rates of the remaining age-sex groups--which is to be expected (see trend (1) and (4) above)--is probably exaggerated. 1/ Accordingly, it was found advisable to assume that the 1951 participation rates for these age-sex groups apply to 1956. 2/

1/ The age-sex groups "15 through 19" show an unexpected rise which may be due to conditions peculiar to New Caledonia.

2/ This analysis of the probable trends in the age-sex participation rates of New Caledonia's non-indigenous population may be contradicted by facts not available to the writer. However, its usefulness is intended mainly to be in indicating the type of analysis that may be necessary in projecting labor force age-sex participation rates. Note that because of the expected inaccuracy of the data, straight line projection was not found advisable.

WORKSHEET 27

PROJECTING AGE-SEX PARTICIPATION RATES TO 1956, NEW CALEDONIA NON-INDIGENOUS POPULATION

	MALE			FEMALE		
	Participation Rates		Projected 1/ to 1956	Participation Rates		Projected 1/ to 1956
AGE GROUP	Census 1946	Census 1951		Census 1946	Census 1951	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Under 15	0.002	0.002	0.002	0.001	0.001	0.001
15 through 19	0.399	0.477	0.477	0.140	0.147	0.147
20 " 24	0.890	0.867	0.867	0.281	0.225	0.225
25 " 29	0.869	0.883	0.883	0.217	0.227	0.227
30 " 34	0.908	0.883	0.908	0.251	0.234	0.251
35 " 39	0.939	0.879	0.939	0.258	0.235	0.258
40 " 44	0.925	0.860	0.925	0.288	0.236	0.288
45 " 49	0.908	0.843	0.908	0.255	0.217	0.255
50 " 54	0.902	0.811	0.902	0.250	0.214	0.250
55 " 59	0.849	0.761	0.849	0.254	0.245	0.254
60 " 64	0.790	0.650	0.650	0.215	0.159	0.159
65 " 69	0.720	0.516	0.516	0.238	0.113	0.113
70 and over	0.488	0.375	0.375	0.137	0.119	0.119

1/ Methods discussed in text.

PROJECTED LABOR FORCE BY AGE AND SEX

Once age-sex participation rates have been projected to the pertinent date (1956 in this case), it is essential to obtain an estimate of the size of the respective age-sex groups of the given population for that same date. This may be obtained by methods described in Part II, Chapters III and IV.

In Worksheet 28, assume that the New Caledonia non-indigenous population was projected to 1956, and the result shown in Columns (2) and (5). In Columns (3) and (6) is listed opposite each age-sex group the corresponding 1956 projected participation rate [obtained from Worksheet 27, Columns (4) and (7)]. 1/ Finally, Columns (4) and (7) are obtained by multiplying each projected age-sex group by its projected participation rate, thus obtaining an estimate of the 1956 labor force by age and sex. The projected total labor force may now be obtained by adding the totals of Columns (4) and (7).

1/ The figures in the last row, however, are obtained by dividing, for the same row, Column (4) by Column (2) for males and Column (7) by Column (5) for females. These show the estimated proportion of total population that will be in the labor force.

WORKSHEET 28

COMPUTING THE LABOR FORCE BY AGE AND SEX NEW CALEDONIA, 1956 NON -INDIGENOUS POPULATION

	MALE			FEMALE		
AGE GROUP	Projected Population 1956	Projected Part. Rates 1956	Projected Labor Force 1956	Projected Population 1956	Projected Part. Rates 1956	Projected Labor Force 1956
(1)	(2)	(3)	(4)=(2)x(3)	(5)	(6)	(7)=(5)x(6)
Under 15	3,647	0.002	7	3,765	0.001	4
15 through 19	942	0.477	449	961	0.147	141
20 " 24	994	0.867	862	843	0.225	190
25 " 29	485	0.883	428	818	0.227	186
30 " 34	500	0.908	454	767	0.251	193
35 " 39	768	0.939	721	715	0.258	184
40 " 44	750	0.925	694	741	0.288	213
45 " 49	588	0.908	534	576	0.255	147
50 " 54	510	0.902	460	472	0.250	118
55 " 59	381	0.849	323	388	0.254	99
60 " 64	279	0.650	181	269	0.159	43
65 " 69	207	0.516	107	240	0.113	27
70+	299	0.375	112	325	0.119	39
Total	10,350	0.5152 1/	5,332	10,880	0.1456 1/	1,584

1/ For illustration only.

NOTE

If population and labor force data in the censuses are known to be of doubtful accuracy, it may be advisable to combine age-sex groups into broader categories than the usual five years. Through such a combination, errors are hoped to balance so that the resulting age-sex participation rate may show more consistency over time. However, the combined five-year age-sex groups must be those showing approximately the same participation rates. Therefore, it may be advisable to use the following broad categories for each sex. (1) "Under 15" or "10 through 14"; (2) "20 through 34"; (3) "35 through 55"; (4) "55 through 65" and (5) "65 and over".

PART IV

A CASE STUDY: THE FIJI ISLANDS

In this handbook, techniques of estimating and projecting population and population characteristics and of estimating and projecting the labor force have been discussed. In many instances, alternative techniques have been introduced for the estimation of a given parameter with the hope that at least one of the techniques will suit the analyst's background training and the basic data available to him. In this final part of the handbook, a brief presentation of all the variables already discussed will be made in order to give the reader a picture of the whole rather than the parts. In order to do this, only one method will be used for computing each parameter. Consequently, many of the techniques expounded previously are neglected in this section.

The area chosen for this task is the Fiji Islands. Estimates and projections of its demographic and labor force variables are undertaken in the following order:

(1) Estimates of the Growth Components of Population Change between the two Census Years 1946 and 1956 (from Part II, Chapter II).

(2) Projection of population by age and sex to 1961 and 1966 with the use of the 1946-1956 "Discount Ratio's" (from Part II, Chapter IV).

(3) Estimates of the Number of Children Reaching Eight Years of Age Annually between 1956 and 1966 (Part II, Chapter V).

(4) Estimating the Labor Force in 1956 and Projecting it to 1966, by Sex and Broad Age Groups (Part III, Chapter I).

Needless to repeat, the computations are undertaken for illustrative purposes only. Furthermore, the exposition will be made mainly in terms of worksheets.

(1) Estimates of The Growth Components of Population Change, 1946 - 1956

Worksheet 29, (Male), Worksheet 30, (Female)

WORKSHEET 29

ESTIMATES OF THE GROWTH COMPONENTS OF POPULATION CHANGE, FIJI ISLANDS 1946 - 1956

MALES

Age Group	Population 1946	10-Year Survival Ratios 1/	Population Excl. Migration 1956	Population 1956	Migration 1946-1956
(1)	(2)	(3)	(4) = (2) x (3)	(5)	(6) = (5) - (4)
Born 1951 - 56	36,061 2/	0.9290	-	-	-
Born 1946-1951	30,380 2/	0.9044	-	-	-
Less than 5	23,520 3/	0.9642	33,501	33,335 3/	-166
5 through 9	19,905	0.9757	27,476	26,679	-797
10 " 14	16,021	0.9748	22,678	22,452	-226
15 " 19	13,072	0.9765	19,421	18,312	-1,109
20 " 24	10,776	0.9670	15,617	15,133	-484
25 " 29	9,907 4/	0.9579	12,765	12,667	-98
30 " 34	8,113 4/	0.9459	10,420	10,167	-253
35 " 39	6,624	0.9252	9,490	9,299	-191
40 " 44	4,928	0.8929	7,674	7,702 4/	+28
45 " 49	4,622	0.8487	6,129	5,960	-169
50 " 54	4,339 4/	0.7856	4,400	4,285	-115
55 " 59	4,593 4/	0.7040	3,923	3,467	-456
60 " 64	4,476 4/	0.6084	3,409	3,391 4/	-18
65 " 69			3,233	2,679	-554
70 " 74	6,196	0.3663	2,723	1,914	-809
75 and over			2,270	2,088	-182
Total (Excluding first two rows)	137,092		185,129	179,530	-5,599

1/ Obtained from Table XIII in Section C, Chapter VI.

2/ Since the censuses were taken around the beginning of the tenth month of 1946 and 1956 (10/2/46 and 9/27/56) it was decided to take 1/6 of the births in the year 1946, all the births in 1947 through 1950, and 5/6 of the births in 1951 as births between 1946 and 1951. Similarly, for the births between 1951 and 1956, 1/6 of the births in 1951, all the births in 1952 through 1955 and 5/6 of the births in 1956 were used. Because of the unavailability of basic statistics, an arbitrary 8% underregistration of births was assumed. For methods of evaluating and adjusting birth data, see Section E, Chapter VI.

Since births in the years 1946, 1947 and 1948 were inclusive of stillbirths and were not broken down to their sex components and we are interested in live births only by sex, the proportion of stillbirths to total births in each of these years was assumed to be equal to the average for the years 1949 through 1952 for which data were available. This rate was thus applied to reported births in 1946, 1947 and 1948 and the results subtracted from the reported birth values. The distribution of these births among the two sexes was then taken to be 51 percent to 49 percent in favor of males.

3/ Assumed 5 percent under enumeration of children (see Section D, Chapter VI).

4/ Adjusted by smoothing census data (see Section D, Chapter VI).

5/ Persons of unknown ages were negligible and thus excluded from total.

WORKSHEET 30

ESTIMATING THE GROWTH COMPONENTS OF POPULATION CHANGE, FIJI ISLANDS 1946 - 1956

FEMALES

Age Group	Population 1946	10-Year Survival Ratios 1/	Population Excl. Migration 1956	Population 1956	Migration 1946-1956
(1)	(2)	(3)	(4) = (2) x (3)	(5)	(6) = (5) - (4)
Born 1951 - 56	34,214 2/	0.9400	-	-	-
Born 1946 - 51	28,686 2/	0.9165	-	-	-
Less than 5	22,997 3/	0.9663	32,161	32,544 3/	+383
5 through 9	19,480	0.9774	26,291	25,795	-496
10 " 14	15,171	0.9633	22,222	21,642	-580
15 " 19	12,356	0.9533	19,040	18,162	-878
20 " 24	11,031	0.9460	14,614	14,776	+162
25 " 29	9,985	0.9389	11,779	12,105	+326
30 " 34	7,568	0.9312	10,435	9,757	-678
35 " 39	5,472	0.9206	9,375	8,749	-626
40 " 44	4,439	0.9056	7,047	6,775	-272
45 " 49	3,459	0.8767	5,038	4,898	-140
50 " 54	3,324 4/	0.8040	4,020	3,733	-287
55 " 59	2,841 4/	0.6929	3,033	2,875	-158
60 " 64	2,298 4/	0.5510	2,672	2,454 4/	-218
65 " 69 -			1,969	1,671	-298
70 " 74 --	2,787	0.3063	1,266	1,247	-19
75 and over -			854	1,327	+473
Total (Excluding first two rows)	123,208 5/		171,816	168,510	-3,306

1/ Obtained from Table XIV in Section C, Chapter VI.

2/ Since the censuses were taken around the beginning of the tenth month of 1946 and 1956 (10/2/46 and 9/27/56) it was decided to take 1/6 of the births in the year 1946, all the births in 1947 through 1950, and 5/6 of the births in 1951 as births between 1946 and 1951. Similarly, for the births between 1951 and 1956, 1/6 of the births in 1951, all the births in 1952 through 1955 and 5/6 of the births in 1956 were used. Because of the unavailability of basic statistics, an arbitrary 8% underregistration of births was assumed. For methods of evaluating and adjusting birth data, see Section E, Chapter VI.

Since births in the years 1946, 1947 and 1948 were inclusive of stillbirths and were not broken down to their sex components and we are interested in live births only by sex, the proportion of stillbirths to total births in each of these years was assumed to be equal to the average for the years 1949 through 1952 for which data were available. This rate was thus applied to reported births in 1946, 1947 and 1948 and the results subtracted from the reported birth values. The distribution of these births among the two sexes was then taken to be 51 percent to 49 percent in favor of males.

3/ Assumed 5 percent under enumeration of children. Thus this figure is equal to that reported in the census divided by 0.95. (See Section D, Chapter VI).

4/ Adjusted by smoothing census data (see Section D, Chapter VI).

5/ Persons of unknown ages were negligible and thus excluded from total.

Column (1): List age groups in five-year brackets. Reserve first two rows for Born 1951-56 and Born 1946-51 respectively.

Column (2): List population in 1946 by five-year age groups and adjust (if necessary) as indicated. In the first two rows list live births (adjusted).

Column (3): List ten-year survival ratios for each five-year age group.

Column (4): For each row, multiply column (2) by column (3) and place two rows below in column (4).

Column (5): List population 1956 adjusted (if necessary) as indicated.

Column (6): For each row subtract column (4) from column (5).

The following must be noted:

1. (4) - (2) = natural increase (N_i)
2. (5) - (4) = (6) = total net migration (M)
3. (5) - (2) = Population change (ΔP)

Thus:

$$\Delta P = N_i + M$$

(2a) Projection of Population to 1966 By Age and Sex

Worksheet 31, (Male), Worksheet 32, (Female)

WORKSHEET 31

PROJECTIONS OF POPULATION TO 1966 WITH THE USE OF DISCOUNT RATIOS MALES

Age Group (1)	Population 1946 (2)	Population 1956 (3)	10-Year Discount Ratios (4) 1/	Population 1966 (5) 2/
Born next 5-9 Yrs.	36,061 3/	46,993 4/	0.9244	-
Born next 0-4 Yrs.	30,380 3/	39,981 4/	0.8782	-
Less than 5	23,520	33,335	0.9546	43,440
5 through 9	19,905	26,679	0.9200	35,111
10 " 14	16,021	22,452	0.9446	31,822
15 " 19	13,072	18,312	0.9690	24,545
20 " 24	10,776	15,133	0.9435	21,208
25 " 29	9,907	12,667	0.9386	17,744
30 " 34	8,113	10,167	0.9493	14,278
35 " 39	6,624	9,299	0.8998	11,889
40 " 44	4,928	7,702	0.8695	9,652
45 " 49	4,622	5,960	0.7501	8,367
50 " 54	4,339	4,285	0.7815	6,697
55 " 59	4,593	3,467	0.5833	4,471
60 " 64	4,476	3,391	0.4276	3,349
65 " 69 -		2,679 -		2,022
70 " 74 --	6,196	1,914 --	0.3370 5/	1,450
75 and over -		2,088 -		2,251 6/
Total (excluding first two rows)	137,092	179,530		238,296

1/ Column (4) entry for a line = column (3) entry two lines below ÷ column (2) entry for the line being computed.

2/ Column (5) entry for a line = column (3) entry two lines above x column (4) entry two lines above the line being computed.

Footnotes continued on Worksheet 32.

WORKSHEET 32

PROJECTIONS OF POPULATION TO 1966 WITH THE USE OF DISCOUNT RATIOS FEMALES

Age Group (1)	Population 1946 (2)	Population 1956 (3)	10-Year Discount Ratios (4) 1/	Population 1966 (5) 2/
Born next 5-9 Yrs.	34,214 3/	44,460 4/	0.9512	-
Born next 0-4 Yrs.	28,686 3/	38,010 4/	0.8992	-
Less than 5	22,997	32,544	0.9411	42,290
5 through 9	19,480	25,795	0.9323	34,179
10 " 14	15,171	21,642	0.9740	30,627
15 " 19	12,356	18,162	0.9797	24,049
20 " 24	11,031	14,776	0.8845	21,079
25 " 29	9,985	12,105	0.8762	17,793
30 " 34	7,568	9,757	0.8952	13,069
35 " 39	5,472	8,749	0.8951	10,606
40 " 44	4,439	6,775	0.8410	8,734
45 " 49	3,459	4,898	0.8312	7,831
50 " 54	3,324	3,733	0.7383	5,698
55 " 59	2,841	2,875	0.5882	4,071
60 " 64	2,298	2,454	0.5426	2,756
65 " 69	2,787	1,671	0.4761 5/	1,691
70 " 74		1,247		1,332
75 and over		1,327		2,021 6/
Total (excluding first two rows)	123,208	168,510		227,826

Footnotes are the same as those in Worksheet 31.

3/ See footnote 2 of Worksheet 29.

4/ Projected sex-age adjusted births. For the detail computations for male births see Worksheets 14 and 15 and Table XII in Section B, Chapter VI.

5/ Obtained by dividing the number "75 and over" in 1956 by the number "65 and over" in 1946.

6/ Obtained by multiplying the discount ratio for "65 and over," column (4) by the number of persons 65 and over in 1956 [total of "65 through 69", "70 through 74" and "75 and over", in column (3)].

Column (1): Same as column (1) in Worksheet 29 or 30.

Column (2): Same as column (2) in Worksheet 29 or 30.

Column (3): Same as column (5) in Worksheet 29 or 30.

Column (4): Divide the size of each age group in 1956, column (3), by the figure two rows above in column (2) and place the result in the latter row in column (4).

Column (5): For each row multiply column (3) by column (4) and place the result two rows below.

(2b) Projections of Population to 1961 By Age and Sex

Worksheet 33, (Male), Worksheet 34, (Female)

WORKSHEET 33

PROJECTIONS OF POPULATION TO 1961 WITH THE USE OF DISCOUNT RATIOS MALES

Age Group (1)	Population 1946 (2)	Population 1956 (3)	5-Year Discount Ratios (4)	Population 1961 (5) 2/
Born next 5-9 Yrs.	36,061	46,993	-	-
Born next 0-4 Yrs.	30,380	39,981	0.9391 1/	-
Less than 5	23,520	33,335	0.9773	37,546
5 through 9	19,905	26,679	0.9600	32,578
10 " 14	16,021	22,452	0.9723	25,612
15 " 19	13,072	18,312	0.9845	21,830
20 " 24	10,776	15,133	0.9718	18,028
25 " 29	9,907	12,667	0.9693	14,706
30 " 34	8,113	10,167	0.9747	12,278
35 " 39	6,624	9,299	0.9499	9,910
40 " 44	4,928	7,702	0.9348	8,833
45 " 49	4,622	5,960	0.8751	7,200
50 " 54	4,339	4,285	0.8908	5,216
55 " 59	4,593	3,467	0.7917	3,817
60 " 64	4,476	3,391	0.7138	2,745
65 " 69		2,679		2,420
70 " 74	6,196	1,914	0.6685	4,466
75 and over		2,088		
Total (excluding first two rows)	137,092	179,530		207,185

1/ Note that it is theoretically justifiable to use the discount ratio for "born next 5-9 years" of Worksheet 31 here instead of this one.

2/ Column (3) line above x Column (4) line above.

WORKSHEET 34

PROJECTIONS OF POPULATION TO 1961 WITH THE USE OF DISCOUNT RATIOS FEMALES

Age Group (1)	Population 1946 (2)	Population 1956 (3)	5-Year Discount Ratios (4)	Population 1961 (5)
Born next 5-9 Yrs.	34, 214	44, 460	-	-
Born next 0-4 Yrs.	28, 686	38, 010	0.9496 1/	-
Less than 5	22, 997	32, 544	0.9706	36, 094
5 through 9	19, 480	25, 795	0.9662	31, 587
10 " 14	15, 171	21, 642	0.9870	24, 923
15 " 19	12, 356	18, 162	0.9899	21, 361
20 " 24	11, 031	14, 776	0.9423	17, 979
25 " 29	9, 985	12, 105	0.9381	13, 923
30 " 34	7, 568	9, 757	0.9476	11, 356
35 " 39	5, 472	8, 749	0.9476	9, 246
40 " 44	4, 439	6, 775	0.9205	8, 291
45 " 49	3, 459	4, 898	0.9156	6, 236
50 " 54	3, 324	3, 733	0.8692	4, 485
55 " 59	2, 841	2, 875	0.7941	3, 245
60 " 64	2, 298	2, 454	0.7713	2, 283
65 " 69		1, 671		1, 893
70 " 74	2, 787	1, 247	0.7381	3, 133
75 and over		1, 327		-
Total (Excluding First two rows)	123, 208	168, 510		196, 035

1/ Note that it is theoretically justifiable to use the discount ratio for "born next 5 - 9 years" of Worksheet 32 here instead of this one.

In Part II, Chapter IV, a short-cut method for projections over ten year spans was introduced. In what follows, the same short-cut method for making a five-year projection of the population by age and sex will be used. The two cases are basically the same except for the discount ratios which now refer to a period of five years rather than ten. In order to obtain these ratios, use the following formula:

$$R_5 = \frac{1 + R_{10}}{2}$$

Where R_5 is the five-year discount ratio and R_{10} the corresponding ten-year ratio in Worksheet 32. For example, the ten-year discount ratio for the female age group "15 through 19" is 0.9797. Thus $R_{10} = 0.9797$ and the five-year ratio for that age group is:

$$R_5 = \frac{1 + 0.9797}{2} = 0.9899$$

(3) Estimates of the Number of Persons Reaching Eight Years of Age Annually October 1956 - October 1966

WORKSHEET 35

WORKSHEET 35

ESTIMATES OF THE NUMBER OF PERSONS REACHING THE AGE OF EIGHT, ANNUALLY

(October 1956 - October 1966)

Year of Birth	Number of Births	Discount Ratio	Number Eight Years	Year Reaching Eight
(1)	(2)	(3)	(4)=(2)x(3)	(5)
1947	10,840	0.9241	10,017	1956
1948	10,881	0.9227	10,040	1957
1949	11,406	0.9213	10,508	1958
1950	11,454	0.9200	10,538	1959
1951	12,334	0.9186	11,330	1960
1952	12,865	0.9172	11,800	1961
1953	13,159	0.9158	12,051	1962
1954	13,090	0.9144	11,969	1963
1955	13,908	0.9131	12,699	1964
1956	14,717	0.9117	13,417	1965
1957	14,816	0.9103	13,487	1966

Column (1): List the years of birth by subtracting nine from each of the years 1956 through 1966.

Column (2): For each year list the total number of adjusted births occurring between October of that year and October of the next year. If as in this case births are reported by one year unit interval, take 3/12 of the births during the listed year plus 9/12 of the births during the next year as representing the births for the year listed. For example, adjusted births in 1947 were 10,301 and in 1948, 11,020. Opposite 1947 in Worksheet 35, list number of births equal to 3/12 (10,301) + 9/12 (11,020) = 10,840.

Column (3): (a) With the help of the Sprague Multipliers determine the number of persons eight years old in the age group "5 through 9" in 1956 and 1966. The total number in these age groups may be obtained from Worksheet 31, columns (3) and (5). (b) Divide the results by the first and last rows of column (2) (in Worksheet 35) respectively and place the results in the first and last rows of column (3). (c) For the remaining rows of column (3) interpolate (arithmetically in this case) between the values in the first and last rows.

Column (4): For each row multiply column (2) by column (3).

Column (5): List the years for which the values in column (4) apply.

(4a) Estimating The Labor Force in 1956 By Sex And Broad Age Groups

WORKSHEET 36

WORKSHEET 36

ESTIMATING THE LABOR FORCE BY SEX AND BROAD AGE GROUP - 1956

Age Group	Population 1956	Participation Rates 1956	Labor Force 1956
(1)	(2)	(3)	(4)=(2)x(3)
		Male	
15 through 19	18,312	0.782	14,320
20 " 64	72,071	0.965	69,549
65 and over	6,681	0.573	3,828
Total	97,064	0.9035 1/	87,697
		Female	
15 through 19	18,162	0.060	1,090
20 " 64	66,122	0.058	3,835
65 and over	4,245	0.017	72
Total	88,529	0.0564 1/	4,997

1/ Over-all participation rate, obtained by dividing column (4) by column (2).

Column (1): List age groups in broad categories.

Column (2): List 1956 population [adjusted as in Worksheet 29, column (5)] also in broad categories.

Column (3): List participation rates from the 1956 census.

Column (4): For each row multiply column (2) by column (3).

(4b) Projecting The Labor Force To 1966 By Sex And Broad Age Groups

WORKSHEET 37

WORKSHEET 37

PROJECTING THE LABOR FORCE BY SEX AND BROAD AGE GROUPS TO 1966

Age Group	Population 1966	Participation Rates 1956	Labor Force 1966
(1)	(2)	(3)	(4)=(2)x(3)
		Male	
15 through 19	24,545	0.782	19,194
20 " 64	97,655	0.965	94,237
65 and over	5,723	0.573	3,279
Total	127,923	0.9123	116,710
		Female	
15 through 19	24,049	0.060	1,443
20 " 64	91,637	0.058	5,315
65 and over	5,044	0.017	86
Total	120,730	0.0567	6,844

Same basic procedure as in Worksheet 36 except that population and participation rates are projections. Projected population may be obtained from Worksheet 31, column (5). Except for total, the participation rates were projected by holding constant the 1956 rates of Worksheet 36, column (2). The participation rate for total is computed by dividing the total of column (4) by the total of column (2).

PART V

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